Heterogeneous Agent Macroeconomics
Eight Lessons and a Challenge

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First part based on joint work with:
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Plan

1. 8 lessons
   - 4 lessons for monetary policy
   - 4 lessons for fiscal policy

2. A challenge
   - HA models with aggregate risk: what we're doing makes no sense and the problem is rational expectations about equilibrium prices!
   - Challenge = what should replace RE? Spell out some criteria.

Heterogeneous agent macroeconomics

- Approach: study macro questions in terms of distributions of micro variables rather than just aggregates
  - typical example: distributions of income and wealth
- Attractive for two reasons
  - conceptually: integrated approach to macro and distribution
  - empirically: integrated approach to micro and macro data

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- Attractive for two reasons
  - conceptually: integrated approach to macro and distribution
  - empirically: integrated approach to micro and macro data
- Typical feature: rich interaction

$$\text{distribution} \quad \Longleftrightarrow \quad \text{macroeconomy}$$

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    - typical example: distributions of income and wealth
- Attractive for two reasons
    - conceptually: integrated approach to macro and distribution
    - empirically: integrated approach to micro and macro data
- Put differently:

macroeconomy is a distribution

Today: HA models for monetary and fiscal policy

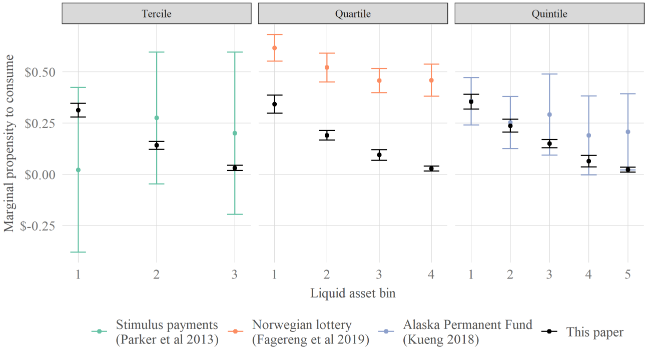
- HANK = Heterogeneous Agent New Keynesian model
- Flesh out predictions and contrast with more traditional approaches:
    - RANK = Representative Agent New Keynesian model
    - Old Keynesian models (Keynesian cross, IS-LM)

Background readings on heterogeneous agent models

- Kaplan, Moll and Violante IMF F&D magazine piece on HANK models
  https://www.imf.org/en/Publications/fandd/issues/2023/03/modern-monetary-policy-kaplan-moll-violante
- Recent New York Times and Financial Times articles on HANK models
  https://www.nytimes.com/2023/07/10/opinion/economic-modeling-hank-representative-agent.html
  https://www.ft.com/content/e0b2b64d-9ab0-4318-b753-3b981299f3ae
- Sargent "HAOK and HANK Models" http://www.tomsargent.com/research/HAOK_HANK.pdf
- Ch.18 "Heterogeneous agent macroeconomics: origins, progress & challenges", Carlin & Soskice 2023 "Macroeconomics: Institutions, Instability & Inequality"
- Cherrier, Garcia-Duarte and Saïdi (2022) "Household Heterogeneity in Macroeconomic Models: A Historical Perspective"
  https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4250570
- Janet Yellen speech https://www.federalreserve.gov/newsevents/speech/yellen20161014a.htm
- BoJ Governor Kuroda speech https://www.boj.or.jp/en/about/press/koen_2017/ko170524a.htm

Marginal Propensities to Consume are large and heterogeneous

Figure 6: Marginal Propensity to Consume by Asset Buffer



Note: This figure compares the estimates of heterogeneity by assets in the passthrough of income shocks to consumption. Parker et al. (2013), Fagereng, Holm and Natvik (2018) and Kueng (2018) use terciles, quartiles, and quintiles respectively. To enable comparability with these prior papers, we calculate the marginal propensity to consume (instead of the elasticity of consumption to income) using their respective bin cutoffs. Our paper, Parker et al. (2013), and Kueng (2018) measure the MPC on nondurables. Fagereng, Holm and Natvik (2018) measures the MPC on total consumption. See Section 3.5 for details.

Source: Ganong-Jones-Noel-Farrell-Greig-Wheat

Household response to monetary policy depends on balance sheets

Households’ consumption response to $r \downarrow$ depends on home ownership status

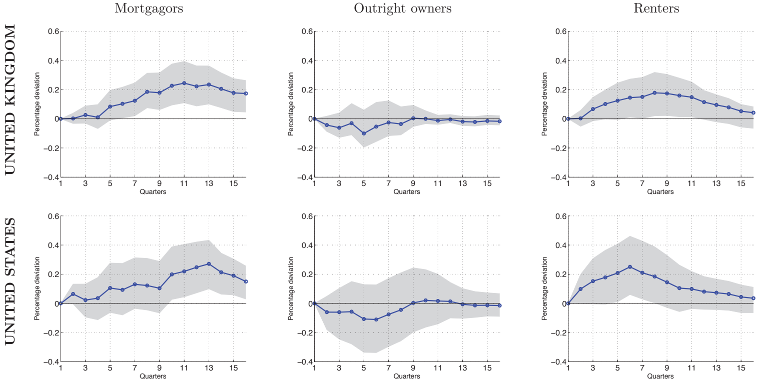


Figure 3: Dynamic effects of a 25 basis point unanticipated interest rate cut on the consumption of non-durable goods and services by housing tenure group. Grey areas are bootstrapped 90% confidence bands. Top row: UK (FES/LCFS data). Bottom row: US (CEX data).

Source: Cloyne-Ferreira-Surico (2018) “Monetary Policy when Households have Debt”

Four lessons for monetary policy

How monetary policy works in RANK

- It's all about intertemporal substitution
- Unconstrained representative agent is on her Euler equation
- Also indirect effects (labor income $\uparrow \Rightarrow$ consumption $\uparrow$) but these are tiny
- In terms of equations:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left[C_{t+1}^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}} \right]$$

or log-linearized version

$$c_t = \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma} \left(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho \right)$$

See e.g. Gali "Monetary Policy, Inflation, and the Business Cycle", Ch.3

What’s wrong with intertemporal substitution logic?

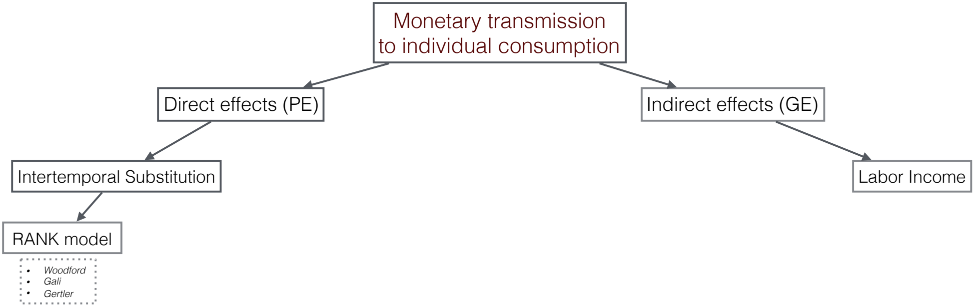
Empirical evidence

- Not everyone is a permanent-income consumer
- Many ‘hand-to-mouth’ with high MPC and low sensitivity to interest rate
- Nontrivial distribution of MPCs across the population

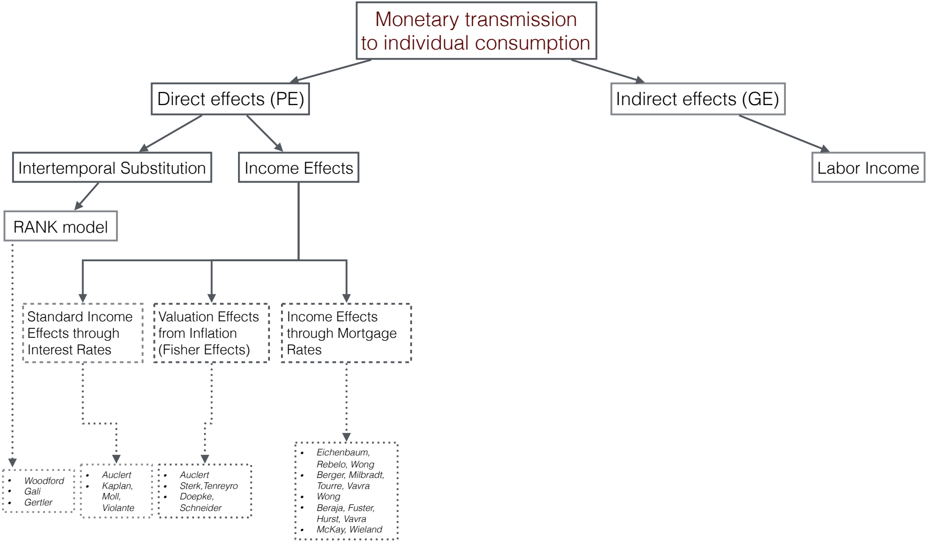
HANK: take this empirical evidence seriously. Implications:

- small direct effects through interest rates
- large indirect/GE effects through labor income
- income/wealth distribution and redistribution matter

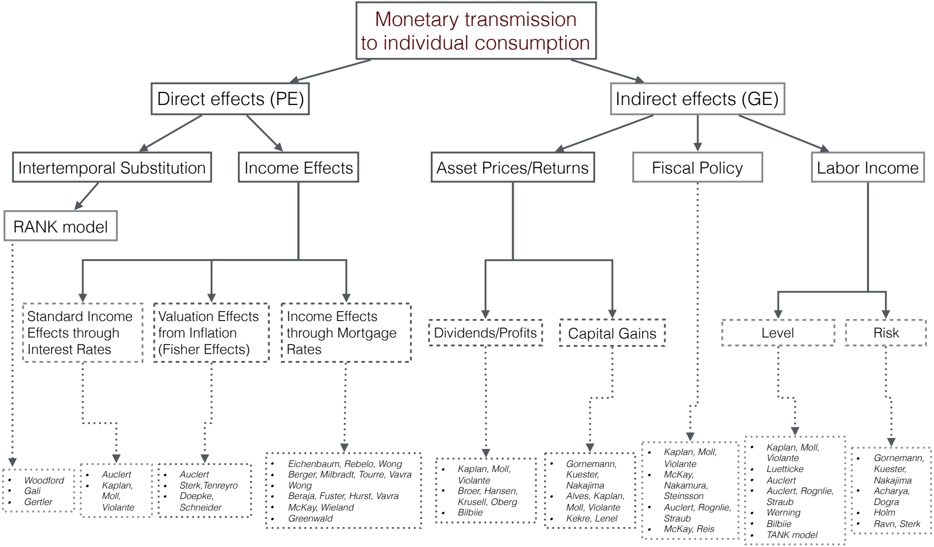
RANK: all about intertemporal substitution (Euler Eqn)



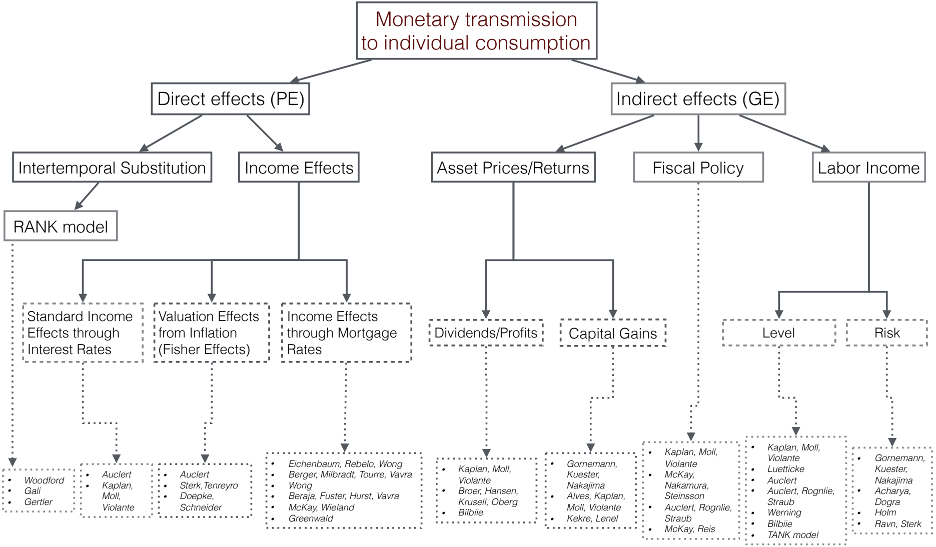
HANK: emphasizes alternative direct effects...



HANK: ... and indirect effects (given high MPCs)



A much richer view of world than rep agent Euler equation



HANK models: four lessons for monetary policy (IMF piece)

- **Lesson 1:** Predicting indirect policy impacts
  - High MPCs $\Rightarrow$ indirect effects $>>$ direct effects
- **Lesson 2:** Some ships are lifted higher, others are sunk
  - Traditional view of monetary policy: "a rising tide raises all ships." This is a fiction.
  - HANK models force us to let go of the fiction that we can cleanly separate stabilization from redistribution
- **Lesson 3:** Fiscal footprints matter
  - Another widespread misconception is the view that monetary policy can be divorced from fiscal policy
  - HANK models reestablish a strong link between the two, showing how monetary policy leaves consequential "fiscal footprints" ($\neq$ FTPL)
- **Lesson 4:** The right tool for redistribution
  - Monetary policy is a blunt tool for redistribution or insurance
  - HANK models tell us that fiscal policy is likely better suited for this task because it can be targeted more precisely to those in need of support

Four lessons for fiscal policy

HANK vs RANK vs Old Keynesian

- Key part of HANK modeling: empirically realistic MPCs
- Interesting implication: model behaviour resembles Keynesian cross, in particular sizable multipliers
- But important difference: micro founded dynamic model, makes precise predictions about behavior as well as inequality, can use it to think about welfare
- Paper that makes these points nicely: "The Intertemporal Keynesian Cross" by Auclert, Rognlie, Straub https://web.stanford.edu/~aauclert/ikc.pdf

HANK models: four lessons for fiscal policy

- **Lesson 1:** The right tool for redistribution or insurance
  - HANK models tell us that fiscal policy *is* the right tool for this task
- **Lesson 2:** Fiscal policy is powerful for aggregate demand management
  - In contrast to RANK pecking order: MP $>>$ FP. Reason = ~~Ricardian equivalence~~
  - Fiscal policy can do anything monetary policy can do (and more)
  - Target stimulus to high MPC households
- **Lesson 3:** Deficit-financed fiscal stimulus is partly self-financing
  - Reason again = ~~Ricardian equivalence~~
  - "Can Deficits Finance Themselves?" by Angeletos-Chen-Wolf
- **Lesson 4:** Insurance rather than stimulus
  - In a recession, the largest welfare gains come not from stimulating aggregate demand but from alleviating hardship
  - Especially true for asymmetric shocks like Covid

The Challenge

or

The Trouble with Rational Expectations
in Heterogeneous Agent Models

Heterogeneous agent models with aggregate risk (HANK = special case)

- Classic papers by Krusell-Smith and Den Haan from late 90s
- Key problem: rational expectations + general equilibrium
  $\Rightarrow$ cross-sectional distribution enters household/firm decision problem
  - true even though households/firms do not really care about distribution and only care about prices
- Recent work: impressive advances solving such models
- ... but this still really holds back HA literature, e.g. ~~non-linearities, crises~~
- My argument in rest of talk:
  - we're spending a lot of intellectual and computational horse power solving a nonsensical problem
  - go back to drawing board and replace RE about equilibrium prices

Plan for rest of talk

1. Back to the roots of RE: it was all about equilibrium prices
2. In het. agent models, RE about equilibrium prices make no sense
3. What should replace RE?

Back to the roots of RE

- Back to John Muth = father of rational expectations (1961 paper)
- ... and to Lucas, Prescott, Sargent & co
- Better modeling expectations of equilibrium prices was the central goal in the development of RE

Muth (1961) “Rational Expectations and the Theory of Price Movements”

3. PRICE FLUCTUATIONS IN AN ISOLATED MARKET

We can best explain what the hypothesis is all about by starting the analysis in a rather simple setting: short-period price variations in an isolated market with a fixed production lag of a commodity which cannot be stored.[5] The market equations take the form

$$
\begin{aligned}
& C_t = -\beta p_t && \text{(Demand)}, \\
(3.1) \qquad & P_t = \gamma p_t^e + u_t, && \text{(Supply)}, \\
& P_t = C_t && \text{(Market equilibrium)},
\end{aligned}
$$

where: P_t represents the number of units produced in a period lasting as long as the production lag,

C_t is the amount consumed,

p_t is the market price in the tth period,

p_t^e is the market price expected to prevail during the tth period on the basis of information available through the $(t-1)$'st period,

u_t is an error term—representing, say, variations in yields due to weather.

All the variables used are deviations from equilibrium values.

In Bob Lucas' words

"[1960s-style macroeconometric models] implied behavior of actual equilibrium prices and incomes that bore no relation to, and were in general grossly inconsistent with, the price expectations that the theory imputed to individual agents." (Lucas 1995, Nobel Lecture)

"One needs a principle to reconcile the price distributions implied by the market equilibrium with the distributions used by agents to form their own views of the future. John Muth noted that [...] these distributions could not differ in a systematic way. His term for this latter hypothesis was rational expectations." (Lucas 1980, "Methods and Problems in Business Cycle Theory")

Lucas and Prescott (1971) "Investment under Uncertainty"

- Paper that first spells out RE the way we now understand it
- Muth: only price means consistent. Lucas-Prescott: whole distributions.

Briefly, we shall be concerned with a competitive industry in which product demand shifts randomly each period, and where factor costs remain stable. In this context, we attempt to determine the competitive equilibrium time paths of capital stock, investment rates, output, and output price for the industry as a whole and for the component firms. From the viewpoint of firms in this industry, forecasting future demand means simply forecasting future output prices. The usual way to formulate this problem is to postulate some forecasting rule for firms, which in turn generates some pattern of investment behavior, which in turn, in conjunction with industry demand, generates an actual price series.

To avoid this difficulty, we shall, in this paper, go to the opposite extreme, assuming that the actual and anticipated prices have the *same* probability distribution, or that price expectations are *rational*.[4] Thus we surrender, in advance, any

$$(9) \qquad v(k, u) = \sup_{x \geqslant 0} \left\{ s(k, u) - x + \beta \int v\left[kh\left(\frac{x}{k}\right), z \right] p(dz, u) \right\}.$$

[4] This term is taken from Muth [**15**], who applied it to the case where the expected and actual price (both random variables) have a common *mean value*. Since Muth's discussion of this concept applies equally well to our assumption of a common *distribution* for these random variables, it seems natural to adopt the term here.

Important goal of developing RE: operational macro theories

Lucas and Prescott (1971) “Investment under Uncertainty”:

- “[By imposing RE], we obtain an operational investment theory linking...”

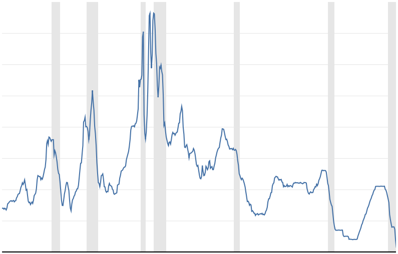
Lucas (1980) “Methods and Problems in Business Cycle Theory”:

- “Our task as I see it [...] is to write a FORTRAN program that will accept specific economic policy rules as ‘input’ and will generate as ‘output’ statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies.”

In HA models, rational expectations about equilibrium prices make no sense

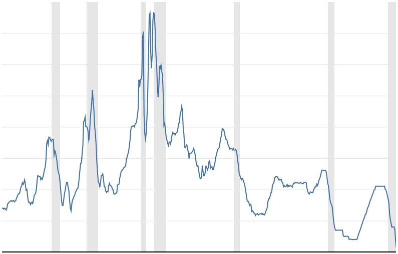
Intuition

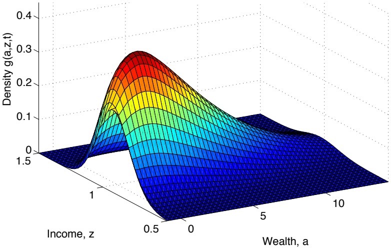
- Suppose I live in one of our models, only care about *r*



Intuition

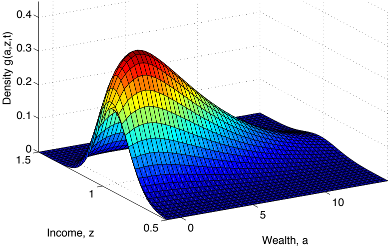
- Suppose I live in one of our models, only care about r
  - I'd realize that in equilibrium r depends on distribution G





Intuition

- Suppose I live in one of our models, only care about r
  - I'd realize that in equilibrium r depends on distribution G
  - RE $\Rightarrow$ in order to forecast r, I'd forecast entire distribution G!



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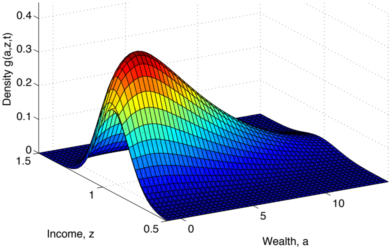

- Makes solution extremely hard (at least for me!)
- But do we really think people do this? I definitely don't!

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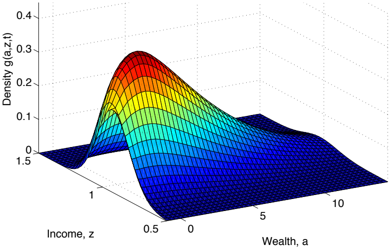

- Makes solution extremely hard (at least for me!)
- But do we really think people do this? I definitely don't!
- Next: explain this in a bit more detail using specific example

Example: forecasting equilibrium w and r

- Start with rep agent economy (RBC model) then add heterogeneity
- Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{T} \beta^t U(c_t, n_t)$$

- Technology:

$$y_t = Z_t F(k_t, \ell_t), \qquad k_{t+1} = x_t + (1-\delta) k_t$$

- Resource constraints:

$$c_t + x_t = y_t, \qquad \ell_t = n_t, \quad \text{all } t$$

- Notes:
  - time horizon T can be finite or ∞. Useful case: two periods $t = 0, 1$
  - aggregate productivity Z_t is stochastic (Markov process)

Representative agent case: competitive equilibrium

Quantities and prices $\{w_t, r_t\}$ such that

1. Households maximize

$$\max_{\{c_t, n_t, a_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{T} \beta^t U(c_t, n_t) \quad \text{s.t.}$$

$$c_t + a_{t+1} = w_t n_t + (1 + r_t) a_t$$

2. Firms maximize

$$\max_{\{x_t, \ell_t, k_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{T} R_{0\to t}^{-1} \left(Z_t F(k_t, \ell_t) - w_t \ell_t - x_t \right) \quad \text{s.t.}$$

$$k_{t+1} = x_t + (1 - \delta) k_t \quad \text{with } R_{0\to t} = \prod_{s=1}^{t} (1 + r_s)$$

3. Markets clear

$$k_t = a_t, \qquad \ell_t = n_t, \quad \text{all } t$$

Key difficulty: households, firms need to forecast w and r

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Focus on wages $\{w_t\}$ for now

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Focus on wages $\{w_t\}$ for now

1. Households maximize

$$\Rightarrow \quad \text{Labor supply} = n(w_t, a_t)$$

2. Firms maximize

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$$k_t = a_t, \qquad \ell_t = n_t, \quad \text{all } t$$

$$\Rightarrow \quad \text{Equilibrium wage} = w(k_t, Z_t)$$

Note: equilibrium is pretty complicated even in this rep agent economy

Solution methods for representative agent case

Two (global) solution methods:

1. Tackle competitive equilibrium directly
   - actually pretty hard even in rep agent case, e.g. (k, K) trick
2. Solve via planning problem
   - no prices so completely sidesteps key difficulty
   - frequent approach in literature (e.g. RBC model)

Heterogeneous agents case: competitive equilibrium

Quantities and prices $\{w_t, r_t\}$ such that

1. Households: heterogeneous in (a_{it}, z_{it}), z_{it} = id. risk, distribution $G_t(a, z)$

$$\max_{\{c_{it}, n_{it}, a_{it+1}\}} \mathbb{E}_0 \sum_{t=0}^{T} \beta^t U(c_{it}, n_{it}) \quad \text{s.t.}$$

$$c_{it} + a_{it+1} = w_t z_{it} n_{it} + (1 + r_t) a_{it}$$

2. Firms (as before): rep firm optimally chooses $\{\ell_t, k_t\}$ given $\{w_t, r_t\}$

3. Markets clear

$$k_t = \int a dG_t(a, z), \qquad \ell_t = \int n_t(a, z) dG_t(a, z), \quad \text{all } t$$

Note: households/firms do not care about dist'n G_t, only care about prices

Key difficulty: households, firms need to forecast w and r

Focus on wages $\{w_t\}$ for now

1. Households: heterogeneous in (a_{it}, z_{it}), z_{it} = id. risk, distribution $G_t(a, z)$

$$\Rightarrow \quad \text{Household } i\text{'s labor supply} = n(w_t, a_{it}, z_{it})$$

2. Firms (as before)

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$$k_t = \int a dG_t(a, z), \qquad \ell_t = \int n_t(a, z) dG_t(a, z), \quad \text{all } t$$

$$\Rightarrow \quad \text{Equilibrium wage} = w(G_t(a, z), Z_t)$$

Note: equilibrium prices depend on entire cross-sectional distribution G_t!

Generic feature of heterogeneous agent models

Rational expectations: forecast prices by forecasting distributions

See this clearly in special case with two time periods $t = 0, 1$

1. Households solve

$$V_0(a, z, G, Z) = \max_{c,n,a'} U(c, n) + \beta\mathbb{E}[V_1(a', z', G', Z')|z, Z] \quad \text{s.t.}$$

$$c + a' = w_0(G, Z)zn + (1 + r_0(G, Z))a$$

$$V_1(a', z', G', Z') = \max_{c',n'} U(c', n') \quad \text{s.t.} \quad c' = w_1(G', Z')z'n' + (1 + r_1(G', Z'))$$

   where G' = cross-sectional distribution at $t = 1$

2. Firm investment decision: similar problem featuring
   - prices $w_1(G', Z')$ and $r_1(G', Z')$
   - value function $J_1(k, G', Z')$

MFG "Monster equation", makes solution extremely hard

Do we really think people do this? I definitely don't!

Solution methods for heterogeneous agent case

1. Linearization or MIT shocks: typical approach in HANK literature
   - certainty (equivalence) for prices so sidesteps key difficulty
   - but not suitable for inflation debate, financial crises, asset pricing, ...

2. Krusell-Smith
   - forecast prices by forecasting moments of distributions, e.g. mean:

$$\bar{a}_t = \int a dG_t(a, z) \quad \text{instead of} \quad G_t(a, z)$$

   - bounded rationality interpretation
   - but do we think people do **that**? I personally also don't

3. Tackling full RE equilibrium: impressive advances in recent literature

(e.g. Schaab, Bilal, Bhandari-Bourany-Evans-Golosov, Han-Yang-E, Gu-Lauriere-Merkel-Payne, Gopalakrishna-Gu-Payne, Huang, Lee, Proehl)

   - nonsensical problem: too much intellectual/computational horse power

Taking stock and what next?

Goal of Muth, Lucas & co when developing RE: operational macro theories

RE achieves exactly this goal in representative agent models

But **RE $\Rightarrow$ het. agent models with aggregate risk "not operational"**

- attributes to people extreme ability to think through equilibrium
- means that people forecast prices by forecasting distributions
- thereby making solution extremely hard

We should go back to drawing board:

- replace RE about equilibrium prices in HA models
- existing attempts (e.g. KS 98) but we need to be more systematic
- Payoff: kill two birds with one stone
    1. make models operational (solution feasible)
    2. ... and more empirically realistic / more interesting

What should replace RE?

What should replace RE?

- I only know the problem, not the solution!
- But spell out some criteria that I find reasonable
- Common element: form expectations about prices directly
  - natural solution
  - different from RE
  - but how discipline prob. distributions to compute price expectations?
- Note: keep RE about non-equilibrium variables, e.g. idiosyncratic z_{it}

Natural solution: form expectations about prices directly

In the 2-period example

$$V_0(a, z, G, Z) = \max_{c,n,a'} U(c, n) + \beta \mathbb{E}[V_1(a', z', G', Z')|z, Z] \quad \text{s.t.}$$
$$c + a' = w_0(G, Z)zn + (1 + r_0(G, Z))a$$
$$V_1(a', z', G', Z') = \max_{c',n'} U(c', n') \quad \text{s.t.} \quad c' = w_1(G', Z')z'n' + (1 + r_1(G', Z'))a'$$

where G' = cross-sectional distribution at $t = 1$

Natural solution: form expectations about prices directly

In the 2-period example

$$V_0(a, z, w, r) = \max_{c,n,a'} U(c, n) + \beta \mathbb{E}[V_1(a', z', w', r')|\cdot] \quad \text{s.t.}$$

$$c + a' = wzn + (1 + r)a$$

$$V_1(a', z', w', r') = \max_{c',n'} U(c', n') \quad \text{s.t.} \quad c' = w'z'n' + (1 + r')a'$$

where expectation $\mathbb{E}$ computed using probability distribution

$$\mathbb{P}(w', r'| \cdot)$$

Natural solution: form expectations about prices directly

In the 2-period example

$$V_0(a,z,w,r) = \max_{c,n,a'} U(c,n) + \beta\mathbb{E}[V_1(a',z',w',r')|\cdot] \quad \text{s.t.}$$

$$c + a' = wzn + (1+r)a$$

$$V_1(a',z',w',r') = \max_{c',n'} U(c',n') \quad \text{s.t.} \quad c' = w'z'n' + (1+r')a'$$

where expectation $\mathbb{E}$ computed using probability distribution

$$\mathbb{P}(w',r'|\cdot)$$

Note: different from Krusell-Smith (forecast prices using moments)

- exception: moment = price

  (e.g. Gomes-Michaelides, Favilukis-Ludvigson-VanNieuwerburgh, Kaplan-Mitman-Violante, Lee-Wolpin, Llull, Storesletten-Telmer-Yaron ...)

Challenge: discipline price expectations $\mathbb{P}$

Price expectations $\mathbb{E}[V(x', p')| \cdot]$ computed using probability distribution

$$\mathbb{P}(p'| \cdot)$$

Challenge: navigating the "wilderness of non-rational expectations"

Sargent (2008) AEA Presidential Address:

- "There is such a bewildering variety of ways to imagine discrepancies between objective and subjective distributions"
- "There is an infinite number of ways to be wrong, but only one way to be correct"
- "Desire to retain discipline of RE" $\Rightarrow$ "cautious modifications of RE"

Three criteria for price expectations $\mathbb{P}$

Price expectations $\mathbb{E}[V(x', p')| \cdot]$ computed using probability dist'n $\mathbb{P}(p'| \cdot)$

Three criteria for $\mathbb{P}$:

1. Simplify solution of het. agent models (make them operational)
   - eliminates models that nest RE: $\mathbb{P}^{\theta}$ with $\mathbb{P}^{\theta=0} = \mathbb{P}^{RE}$ (e.g. diagnostic)
2. Consistency with empirical evidence
   - large literature, e.g. survey expectations
     (e.g. Manski, Armantier-et-al, Weber-DAcunto-Gorodnichenko-Coibion, DAcunto-Weber, Handbook of Economic Expectations)
   - RE = "communism" but huge heterogeneity in data $\Rightarrow \mathbb{P}_i(p')$?
3. (Some) consistency with actual equilibrium prices
   - $\mathbb{P}$ "not too far" from dist'n of actual prices $||\mathbb{P}(p') - \mathbb{P}^{actual}(p')|| < \varepsilon$
   - fixed point problem, expectations respond to policy (Lucas critique)
   - perhaps don't need consistency for entire $\mathbb{P}$, e.g. only $\mathbb{E}[p']$?

Some promising directions and keywords (non-exhaustive)

- Self-confirming equilibrium and least-squares learning
  Bray, Marcet-Sargent, Fudenberg-Levine, Cho-Sargent
- Internal rationality
  Adam-Marcet, Adam-Marcet-Beutel
- Reinforcement learning ($\neq$ neural networks)
  “optimal control of incompletely-known Markov decision processes” (Sutton-Barto)
- “Sequence space” for prices ($\neq$ recursive)
  Boppart-Krusell-Mitman, Auclert-Bardóczy-Rognlie-Straub (but w/o Jacobians)
- Simple models
  Molavi
- Temporary equilibrium (but $\neq$ criterion 3)
  Grandmont, Piazzesi-Schneider
- Cross-domain extrapolation and pessimism bias
  Cenzon, Taubinsky-Butera-Saccarola-Lian, Bordalo-Burro-Coffman-Gennaioli-Shleifer
- ...

All of these: interesting in RA models but potentially larger payoff in HA models

Summary

1. HANK models change how we think about macro policy

- 4 lessons for monetary policy
- 4 lessons for fiscal policy

2. The trouble with RE in heterogeneous agent models

- We spend lots of intellectual and computational horsepower solving nonsensical problem $\Rightarrow$ we should drop RE about equilibrium prices
- Open question: what should replace RE?
- ... how discipline $\mathbb{P}(p'|\cdot)$ to compute price expectations $\mathbb{E}[V(x', p')|\cdot]$?
- Spelled out three criteria for $\mathbb{P}$

Thanks!