# Trade Fragmentation, Inflationary Pressures and Monetary Policy *

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Latest Version: September 2024

**Abstract**

How does trade fragmentation affect inflationary pressures? What is the response of monetary policy needed to sustain inflation at target? To answer these questions, we develop a heterogeneous agent, open-economy model featuring imperfect international risk-sharing. The model captures both the demand and supply side effects of fragmentation. It illustrates how the impact of fragmentation on inflationary pressures and the appropriate policy response depends not only on the direct effect of higher import prices on supply but, crucially, on how aggregate demand adjusts in response to lower real incomes and productivity stemming from fragmentation.

**Keywords:** Monetary policy, trade fragmentation, open economies, inflation, heterogeneity, globalisation
**JEL classification:** F12, F15, F41, F62

*The views expressed in this paper do not represent those of the Bank of England. We are grateful to Raghu Rajan for an insightful discussion, to Francesco Caselli, Sebastian Ellingsen, Rich Harrison and Jan Vlieghe for constructive conversations, and to participants at the 23rd BIS Annual Conference for helpful comments and suggestions. All errors are our own.

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# 1 Introduction

Global trends have shifted noticeably in recent decades. The protracted postwar increase in trade openness has stalled, amidst a resurgence in trade wars and protectionism. This shift is visible in Figure 1, which plots the long-term trajectory of global trade flows relative to world GDP, as well as in Figure 2, which shows the path of a broad index of economic fragmentation since the 1970s (Fernández-Villaverde, Mineyama, and Song, 2024). Both figures show a sharp change in trends starting around the financial crisis, with fragmentation increasing gradually for over more than a decade, before spiking up during the pandemic and Russia's invasion of Ukraine. The geopolitical factors driving these changes are likely to persist. New trade paradigms, such as friendshoring or fragmentation into trading blocs of geopolitically aligned countries are becoming normalized (Yellen, 2022). This reconfiguration of trade patterns raises concerns about potential losses in efficiency and aggregate output (Javorcik, Kitzmueller, Schweiger, and Yıldırım, 2022; Georgieva, 2023).

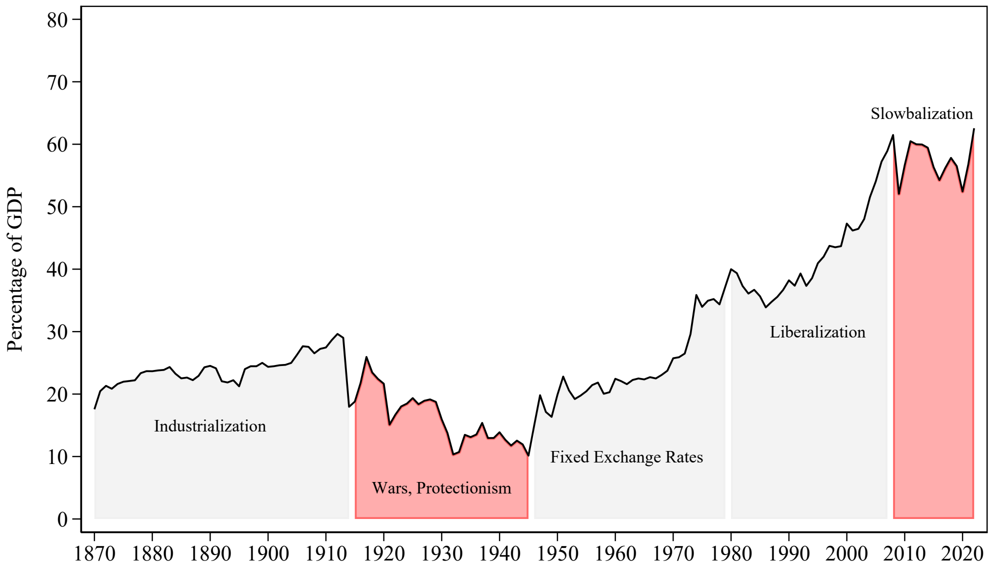
A key question for policymakers is how trade fragmentation will affect inflation dynamics and the optimal monetary policy response. The conventional view suggests that as nations retreat from global integration and supply chains duplicate, production costs will rise, leading to higher inflation (e.g., Lagarde (2023), Goodhart and Pradhan (2020)). Just as the observed disinflationary pressures of the 1990s-2000s coincided with a rapid increase in integration, a reversal of that process is expected to be inflationary. However, this relationship remains contentious. Other forces besides globalisation may have contributed to the era of disinflation, such as advances in manufacturing (IMF (2006)), the shift to inflation-targeting regimes (Roberts (2006)), and the lower bound constraint on interest rates in many countries (Attinasi and Balatti (2021)). Taking the United States as an example, estimates of the disinflationary effects of increased trade integration appear modest (Yellen (2006)), especially considering the presence of a flexible exchange rate, which theoretically shields a country from the direct effects of globalization.[1]

In this paper, we study the effect of trade fragmentation on the macroeconomy. Modelling fragmentation as an increase in the price of imported goods or, alternatively, as a fall in tradable sectors' productivity, we illustrate how the inflationary impact of fragmentation hinges crucially on the adjustment of aggregate demand. Higher import prices or lower productivity in tradable sectors not only constrain supply through higher marginal costs but also demand through lower consumption and real incomes - the general equilibrium effects. Consequently, the net impact on inflationary pressures is a priori ambiguous.

We capture these competing channels in a two-sector, open economy New Keynesian

[1] Kamin, Marazzi, and Schindler (2004), for example, show that the impact of Chinese exports on global prices has been, while non-negligible, fairly modest. Moreover, these studies do not explicitly take into account exchange rate adjustments. During the second half of the 1990s, the dollar experienced a substantial appreciation, driven by heightened investment flows attracted by the prospect of higher productivity growth and increased profits (Kohn (2005)). This might have amplified the downward trend in dollar prices of U.S. imports, further lowering aggregate inflation.

Figure 1: Sum of exports and imports,% of GDP

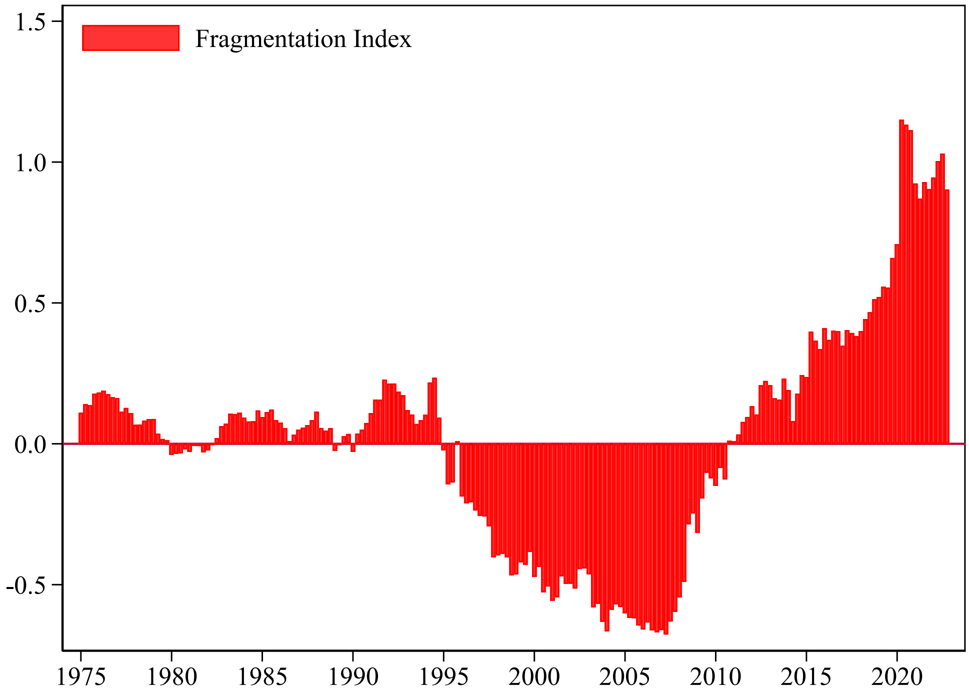



*Source*: Peterson Institute for International Economics; Jordà-Schularick-Taylor Macrohistory Database; Penn World Table (10.0); World Bank; OWID.

model featuring household heterogeneity and home bias in consumption. Specifically, following Debortoli and Galí (2017), the economy features two types of consumers. The first type consists of *unconstrained* agents, with access to security markets. To account for frictions in international financial markets, we introduce imperfect international risk sharing: unconstrained agents can trade risk-free foreign bonds and they face convex costs of holding assets in quantities that deviate from some long-run level (Schmitt-Grohé and Uribe (2003)). The second type consists of *constrained* hand-to-mouth households, who consume only out of their labour income and have no access to financial markets. The domestic economy trades with the rest of the world, importing goods for direct consumption, for use as intermediate inputs, or both.

We consider three scenarios to show how the form in which fragmentation occurs has different macroeconomic implications. First, we consider a gradual (and permanent) increase in the price of imported goods. This yields a persistent increase in imported inflation, which lasts until the import price stabilises at a higher level (in the medium-to-longer term). Aggregate consumption falls in response to fragmentation, as both financially constrained and unconstrained households suffer real income losses: the real disposable income of hand-to-mouth consumers falls as a direct consequence of higher prices, restricting their real spending; in turn, financially unconstrained households, who take into account their permanent-income losses also reduce their consumption in anticipation of lower future incomes. This further accentuates the fall in aggregate demand, spilling over to hand-to-mouth consumers. Real wages fall both because of the negative terms-of-trade effect and because of the fall in domestic demand. Financially constrained households make up for some of the fall in in-

Figure 2: Fragmentation has increased since 2008.




*Source*: Fernández-Villaverde, Mineyama, and Song (2024)

come by increasing their labour supply. The fall in aggregate demand pushes down on domestic inflation. Aggregate CPI (consumer price index) inflation, a composite of domestic and imported goods inflation, falls, given the larger weight of domestic components on the basket. The reduction in demand is reflected in the real natural rate of interest, which decreases with the fragmentation shock. This suggests when demand adjusts, the overall effect is not inflationary. The calibration leads to a long period of stagnation, with low demand and low inflationary pressures; in that setting, monetary policy needs to loosen in order to bring inflation back to target.

Next, we consider a fully front-loaded, permanent increase in the price of imported goods.[2] The shock creates a sharp temporary trade-off, with inflation increasing and aggregate demand falling on impact. Both financially unconstrained and constrained households lower their consumption. The fall in real wages (relative to the price of imported inputs) triggers a labour supply response from the hand-to-mouth consumers. On impact, the short-term real interest rate increases, which requires a tightening in monetary policy to bring inflation back to target. The result is a temporary overshoot in inflation, with longer-term losses in income and consumption.

Finally, we study a fall in the total factor productivity (TFP) of tradable goods, as a po-

[2] This is akin to the recent U.S. and E.U. tariffs on Chinese electric vehicles, reaching up to 100 percent and 38 percent, respectively.

tential consequence of increased fragmentation. The fall is persistent, but not permanent, as TFP reverses over time. Real wages fall, mimicking the fall in TFP. Financially unconstrained consumers can smooth the impact on consumption, but constrained hand-to-mouth households lower their consumption and increase their labour supply in response to lower disposable incomes. Whilst in principle the impact of this shock on the natural real rate is ambiguous, in our calibration, the natural rate falls and demand and supply balance in a way that the shock is not inflationary.

In summary, all three fragmentation scenarios lead to a contraction in aggregate supply. However, they have different implications for the demand for goods and services. Conventional assessments of the impact of fragmentation on inflation often abstract from the demand-side or general-equilibrium impact that fragmentation can have through lower real incomes. While the direct (or partial equilibrium) effect of fragmentation might be inflationary, the general equilibrium effect could dampen inflation, as lower real incomes weigh on aggregate demand. The effects of fragmentation on inflation dynamics and the direction of monetary policy cannot be decoupled from its impact on the natural real interest rate ($r^*$). As trade fragmentation affects the desired levels of savings and spending, the balance between these supply and demand forces ultimately determines the sign and size of changes in the natural rate of interest.

To sharpen our understanding of these dynamics, we vary two key parameters in our simulations: the share of hand-to-mouth agents and the degree of home bias in consumption. A higher share of hand-to-mouth households leads to a smaller fall in consumption on impact. This is because fewer forward-looking households anticipate the adjustment in consumption in response to the fall in their permanent income. The demand adjustment is still sufficient to lower domestic inflationary pressures and offset the increase in imported goods inflation. The extent of home bias in consumption seems to play a more important role. More open economies (with lower home bias) have higher exposure to shocks in foreign prices, which is reflected in the responses of consumption and production. In the scenarios with a persistent increase in foreign prices, whether gradual or front-loaded, we see a deeper fall in the natural rate in the more open economy. This reverts in the case of negative TFP shock; this is because although the shock primarily affects the tradable sector, it is a direct shock to domestic production and consumption, affecting the consumption basket of the more closed economy to a greater extent (a more open economy can diversify away the domestic shock).

To build intuition, we analyse a simple representative-agent New Keynesian (RANK) version of our model as a special case of our TANK baseline model, where there are no constrained households. We also consider an extension of the RANK model with nominal wage rigidities. This friction introduces additional supply-side constraints, leading to a fall in output in both sectors, with implications for the composition of domestic inflationary pressure.

**Related Literature** We build on a rich literature studying monetary policy in small open economies (SOEs), including the seminal work of Benigno and Benigno (2003) and Gali and Monacelli (2005). Other important contributions to this line of research include but are not limited to, Santacreu et al. (2005) and De Paoli (2009), who study tradable and non-tradable sectors in SOEs, and Schmitt-Grohé and Uribe (2003), who introduce imperfect international risk sharing and price stability.

We also draw on an extensive literature that studies the impact of external shocks on macroeconomic outcomes using structural models, such as Romero et al. (2008), Catão and Chang (2013), Hevia and Nicolini (2013), Bergholt (2014), Ferrero and Seneca (2019), Wills (2013), Drechsel, McLeay, and Tenreyro (2019), Broadbent, Di Pace, Drechsel, Harrison, and Tenreyro (2023), and Guerrieri, Marcussen, Reichlin, and Tenreyro (2024). Recent contributions to this literature focus on the transmission of external shocks in models with household heterogeneity. Auclert, Rognlie, Souchier, and Straub (2021) study the real income channel of exchange rate depreciations, highlighting the importance of trade elasticities for the amplification of this channel in a HANK model. The macroeconomic impact of external shocks depends on their effects on relative factor prices. In this respect, this paper is related to papers using structural heterogeneous agent models to understand the impact of the recent energy price shock. In models with labour and imported energy as complementary inputs in production and consumption, Auclert, Monnery, Rognlie, and Straub (2023) and Chan, Diz, and Kanngiesser (2024) show how the demand-side effects of this shock depend on how it redistributes economic resources.[3]

Finally, we build on the vast literature that has examined the macroeconomic effects of globalisation. While increased competition in import prices has placed downward pressure on prices of manufactured goods, studies have shown that globalisation has had a negative, but economically small if not negligible effect on core inflation (Carluccio, Gautier, and Guilloux-Nefussi (2023)). Moreover, there is evidence that global disinflationary forces, such as the shift to inflation targeting regimes (European Central Bank (2021), Roberts (2006), Attinasi and Balatti (2021)) or technological advances in manufacturing (IMF (2006)) could better explain disinflationary forces. Theoretical results provide support for these findings. Sbordone (2008) shows that in a model in which firms' desired markup is a function of its relative market share, an increase in the number of traded goods can generate real rigidities that affect the slope of the Philips curve. As the economy moves to a steady state with higher trade, the elasticity of demand that firms face increases, but the elasticity of the desired markup declines. These two competing forces determine how the inflation-marginal costs component of the Phillips curve slope varies. Using trade data from 1960 to 2006, Sbordone (2008) shows that it remains uncertain whether the trade increase observed during the globalisation era is strong enough to have generated a decline in this component of the

[3]The process of fragmentation, like the green transition, will likely entail a gradual adjustment of relative prices. Del Negro, Di Giovanni, and Dogra (2023) show that climate policies are not necessarily inflationary, as this depends on the relative degree of price stickiness in green and brown sectors.

slope. On the empirical front, Chen, Imbs, and Scott (2009) provide evidence of short-run pro-competitive effects from increased openness. They also show that trade liberalization can have ambiguous effects in the long run, as firms can respond to increased competition by locating to protected markets.

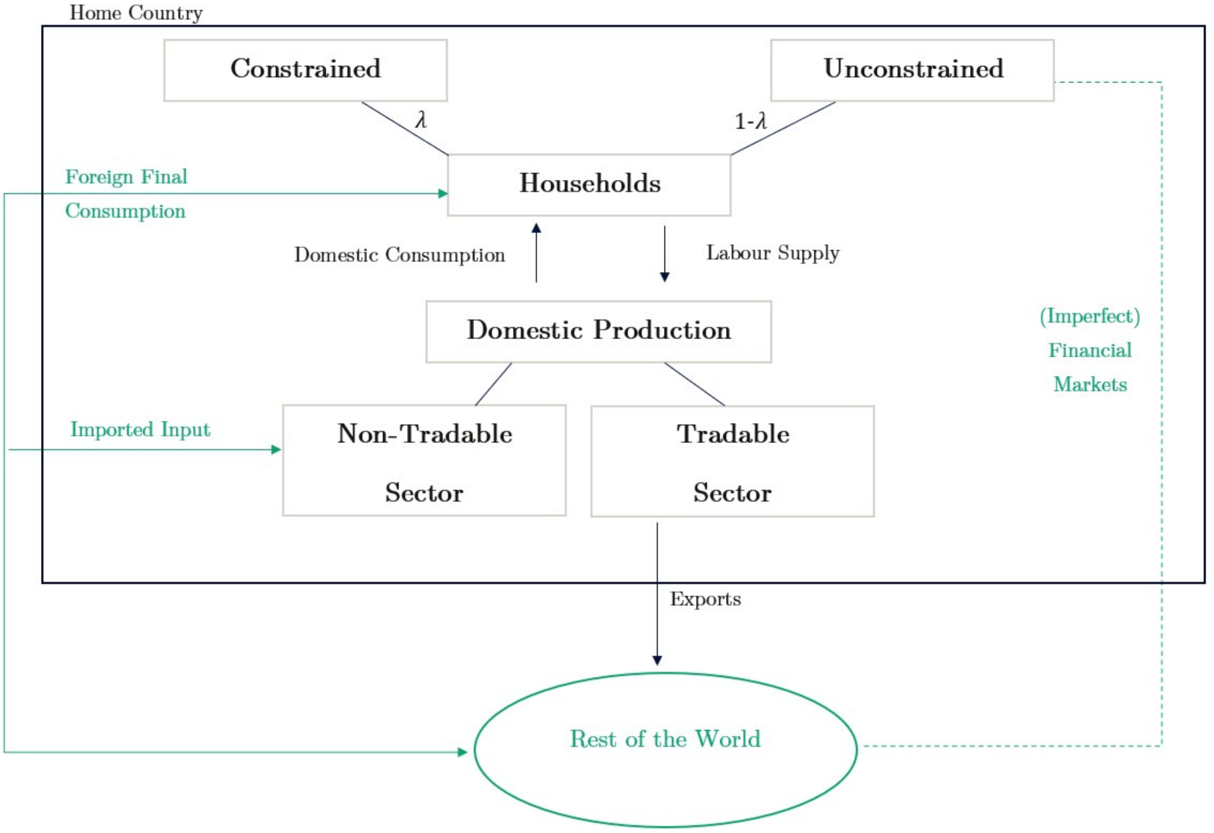
Our paper also relates to the strand of literature pioneered by Rogoff et al. (2003) and Rogoff et al. (2006), which looks at how economic integration affects global inflationary trends. We abstract, however, from the political economy factors studied by Afrouzi, Halac, Rogoff, and Yared (2024), who argue that globalisation would worsen the trade-offs faced by central banks, leading them to succumb to political pressures and deviate from or abandon their inflation targets. The question we ask in this paper is a different one: what would it take for central banks to bring inflation back to target under different fragmentation scenarios? As we show, in some scenarios, activity and inflation both fall, leading to stagnation (that is, without a trade-off); in others, activity and inflation move in opposite directions, creating short-term trade-offs or temporary stagflation. What is required of monetary policy to return inflation to target depends on how aggregate demand responds to lower incomes in general equilibrium. This is contingent on a number of structural parameters that we consider, as well as on the trajectory of fragmentation, particularly on the extent to which the impact on import prices is gradual or front-loaded.

**Outline** Section 2 develops our theoretical framework. Section 3 calibrates the model and analyses shocks that are linked to trade fragmentation. Next, it studies the relative importance of heterogeneity and home bias in the policy response to trade fragmentation. Section 4 studies an extension with nominal wage rigidity. Section 5 presents concluding remarks and potential directions for future research.

# 2 Baseline Model

The goal of this section is to deliver qualitative insights into shocks that relate to deglobalisation. We present a small open economy model that builds on Drechsel, McLeay, and Tenreyro (2019) and Ferrero and Seneca (2019). To capture a more realistic response of aggregate demand to international shocks, we introduce constrained and unconstrained households as in Debortoli and Galí (2017). Finally, to study the impact of fragmentation, we introduce an imported input used in the production of domestic goods.

Figure 3: Model Structure from Home Country Perspective.




## 2.1 Households

There is a continuum of households with identical preferences at any given point in time $t$. They consume $C_t$ and supply labour $N_t$, at wage $W_t$, leading to an expected utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - x_\ell \frac{N_t^{1+\phi}}{1+\phi} \right\}.$$

The parameters $\beta$, $\sigma$, and $\phi$ capture the discount factor, the inverse intertemporal elasticity of substitution and the inverse Frisch elasticity, respectively. $x_\ell$ is the disutility weight put on labour, which we set equal to one in the TANK case. Figure 3 presents an illustration of the model described in this section.

A constant measure (1-$\lambda$) of households are *unconstrained* $U$ and have access to international and domestic financial markets. Their period budget constraint is given by:

$$P_t C_t^U + B_t + \mathcal{E}_t B_t^* = B_{t-1}(1+i_{t-1}) + \mathcal{E}_t B_{t-1}^*(1+i_{t-1}^*) + W_t N_t^U - \frac{\chi}{2}\mathcal{E}_t P_t^* \left( \frac{B_t^*}{P_t^*} - \bar{b}^* \right)^2 \quad (1)$$

where $B_t$ denotes the holdings of a risk-free one-period nominal bond in $H$ currency, which pays the nominal interest rate $i_t$. $B_t^*$ is the risk-free one-period nominal bond in foreign currency, where $i_t^*$ is the foreign interest rate. $\mathcal{E}_t$ is the nominal exchange rate (expressed in domestic relative to foreign currency terms). Following Schmitt-Grohé and Uribe (2003), we assume that there is a quadratic cost in changing the real bond position when trading in the foreign bond market with respect to a real steady-state value, $\bar{b}^*$. These costs are a common feature of small open economy models to ensure that the model returns to a unique steady-state net foreign asset position following a transitory shock. $\chi$ is a non-negative parameter that measures this cost in terms of units of the consumption index. We can rewrite the budget constraint in real terms, for simplicity as:

$$C_t^U + \frac{B_t}{P_t} + \mathcal{S}_t \frac{B_t^*}{P_t^*} = \frac{B_{t-1}}{P_{t-1}} \frac{(1+i_{t-1})}{(1+\pi_t)} + \mathcal{S}_t \frac{B_{t-1}^*}{P_{t-1}^*} \frac{(1+i_{t-1}^*)}{(1+\pi_t^*)} + \frac{W_t}{P_t} N_t^U - \frac{\chi}{2} \mathcal{S}_t \left( \frac{B_t^*}{P_t^*} - \bar{b}^* \right)^2$$

or, introducing real variables, as;

$$C_t^U + b_t + \mathcal{S}_t b_t^* = b_{t-1} \frac{(1+i_{t-1})}{(1+\pi_t)} + \mathcal{S}_t b_{t-1}^* \frac{(1+i_{t-1}^*)}{(1+\pi_t^*)} + w_t N_t^U - \frac{\chi}{2} \mathcal{S}_t \left( b_t^* - \bar{b}^* \right)^2 \tag{2}$$

where $b_t = \frac{B_t}{P_t}, b_t^* = \frac{B_t^*}{P_t^*}, w_t = \frac{W_t}{P_t}$, and $\mathcal{S}_t = \frac{\mathcal{E}_t P_t^*}{P_t}$ is the real exchange rate.

Unconstrained households maximise the expected lifetime utility by choosing a sequence $\{C_t^U, N_t^U, b_t, b_t^*\}_{t=0}^{\infty}$ subject to the sequence of budget constraints (2). The first-order conditions with respect to $C_t^U, N_t^U, b_t, b_t^*$ for the unconstrained agents are respectively given by:

$$\begin{aligned}
(C_t^U)^{-\sigma} &= \delta_t \\
(N_t^U)^{\phi} &= \delta_t \frac{W_t}{P_t} \\
\delta_t &= \beta \mathbb{E}_t \left[ \frac{(1+i_t)}{(1+\pi_{t+1})} \delta_{t+1} \right] \\
\delta_t [\mathcal{S}_t + \mathcal{S}_t \chi (b_t^* - \bar{b}^*)] &= \beta \mathbb{E}_t \left[ \frac{(1+i_t^*)}{(1+\pi_{t+1}^*)} \mathcal{S}_{t+1} \delta_{t+1} \right]
\end{aligned}$$

where $\delta_t$ is the Lagrange multiplier on the budget constraint. The optimality conditions are therefore given by:

$$(N_t^U)^{\phi} = (C_t^U)^{-\sigma} \frac{W_t}{P_t} \tag{3}$$

$$\frac{1}{(1+i_t)} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1}{(1+\pi_{t+1})} \right] \tag{4}$$

$$\left[ 1 + \chi (b_t^* - \bar{b}^*) \right] = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1+i_t^*}{(1+\pi_{t+1}^*)} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} \right] \tag{5}$$

where $\Pi_{t+1} = (1+\pi_{t+1}) = \frac{P_{t+1}}{P_t}$. We define $\Lambda_{t,t+1}^U = \beta \left( \frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma}$ as the relevant stochastic discount factor, given that only the unconstrained households have access to the bonds. The household's optimality condition for labour gives the labour supply relation (3). The first order condition for $b_t$ implies the Euler equation (4). Finally, households' choices of foreign and domestic bonds give rise to an uncovered interest rate parity condition which links the expected exchange rate change to the differential between the domestic and foreign interest rate. The conditions on $b_t$ and $b_t^*$ imply equation (5), the deviation from the uncovered interest-rate parity (UIP). According to equation (5), consumption risk will *not* necessarily be shared internationally at all times, as long as $b_t^* \neq \bar{b}^*$. This is due to convex costs associated with adjusting foreign bond holdings, which reflects frictions in international financial markets. While in steady state $b_t^* = \bar{b}^*$, outside of steady state $\mathcal{D}_t = \left[1 + \chi(b_t^* - \bar{b}^*)\right]$ can fluctuate inefficiently, contingent on shocks.

$$\chi(b_t^* - \bar{b}^*) = \mathbb{E}_t \left[ \Lambda^U_{t,t+1} \left( \frac{(1+i_t^*)}{(1+\pi^*_{t+1})} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} - \frac{(1+i_t)}{((1+\pi_{t+1}))} \right) \right]$$

The remaining $\lambda$ fraction of households are fully *constrained* $C$: they do not have access to financial markets and cannot smooth their consumption over time. Therefore they consume their labour income and transfers each period:

$$P_t C_t^C = W_t N_t^C$$

$$C_t^C = \frac{W_t}{P_t} N_t^C \tag{6}$$

Aggregate consumption is defined as $C_t \equiv (1-\lambda)C_t^U + \lambda C_t^C$. Aggregate labour is $N_t = (1-\lambda)N_t^U + \lambda N_t^C$. Finally, we define the heterogeneity index as the ratio between unconstrained and constrained consumption:

$$\Gamma_t \equiv \frac{C_t^U}{C_t} \tag{7}$$

Total consumption is a CES aggregate of domestic and foreign goods:

$$C_t \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $1-\alpha$ captures the home bias: smaller values of $\alpha$ imply that the economy consumes less foreign goods. However, similarly to Santacreu et al. (2005), home production now includes both *tradable* $T$ and *non-tradable* $N$ goods. Therefore, $C_{H,t}$ is a composite of consumption goods produced in the domestic economy, given by:

$$C_{H,t} = \left[ (1-\gamma)^{\frac{1}{\nu}} C_{N,t}^{\frac{\nu-1}{\nu}} + \gamma^{\frac{1}{\nu}} C_{T,t}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu-1}{\nu}} \tag{8}$$

where $\gamma$ is the share of tradable goods in the economy. The bundles of tradables and non-tradables are given, respectively, by:

$$C_{T,t} \equiv \left( \int_0^1 C_{T,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$C_{N,t} \equiv \left( \int_0^1 C_{N,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon$ is the elasticity of substitution across varieties. We define $C_{F,t}$ in an analogous way. The aggregate CPI price level, $P_t$ and the domestic price level, $P_{H,t}$ are respectively given by:

$$P_t \equiv \left[ (1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$
$$P_{H,t} \equiv \left[ (1-\gamma) P_{N,t}^{1-\nu} + \gamma P_{T,t}^{1-\nu} \right]^{\frac{1}{1-\nu}}$$

Aggregate prices depend on domestic prices and foreign prices according to the home bias of the country. In turn, domestic prices depend on traded and non-traded goods prices. The price index for non-tradable goods, analogously for tradable and foreign goods, is given by:

$$P_{N,t} \equiv \left( \int_0^1 P_{N,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

Total consumption expenditure by households is given by the sum of the expenditures on domestic and foreign goods they consume:

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_{T,t} C_{T,t} + P_{N,t} C_{N,t} + P_{F,t} C_{F,t}$$

The terms of trade are defined as the price of imports in terms of the price of domestic goods.

$$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{9}$$

We assume that the law of one price holds for individual goods at all times so:

$$\mathcal{S}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t} = \mathcal{T}_t^{1-\alpha}$$

where $\mathcal{S}_t$ is the real exchange rate.

The system of demand functions is hence given by:

$$C_{H,t} = (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t$$
$$C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t$$
$$C_{N,t} = (1-\gamma) \left( \frac{P_{N,t}}{P_{H,t}} \right)^{-\nu} C_{H,t}$$
$$C_{T,t} = \gamma \left( \frac{P_{T,t}}{P_{H,t}} \right)^{-\nu} C_{H,t}$$
$$C_{N,t}(i) = \left( \frac{P_{N,t}(i)}{P_{N,t}} \right)^{-\epsilon} C_{N,t}$$

## 2.2 Firms

Households supply labour to both the tradable and non-tradable sectors, such that:

$$N_t = N_{T,t} + N_{N,t} = N_t^C \lambda + N_t^U (1 - \lambda) \tag{10}$$

Labour is completely mobile across sectors, therefore there is only one wage rate in equilibrium. In this version of the model, we do not allow for redistribution of firms' profits to households or across households. Indeed, in the baseline calibration, all real profits $\Psi_t$ are assumed to accrue to foreign households who consume abroad.[4]

### 2.2.1 Domestic Non-tradable Goods sector

**Final Goods Producers**

Competitive final goods producers assemble the intermediate goods $Y_{N,t}(i)$, where $P_{N,t}(i)$ is the price charged by the individual firm $i$. Their optimisation problem is:

$$\max_{Y_{N,t}(i)} \int_0^1 P_{N,t}(i) Y_{N,t},$$

subject to an aggregation technology with constant elasticity of substitution:

$$Y_{N,t} \equiv \left( \int_0^1 Y_{N,t}(i)^{\frac{\epsilon - 1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}}$$

where $P_{N,t}(i)$ is the price charged by the individual firm $i$. This results in a downward-sloping demand function for firm $i$'s product is

$$Y_{N,t}(i) = \left[ \frac{P_{N,t}(i)}{P_{N,t}} \right]^{-\epsilon} Y_{N,t} \tag{11}$$

**Intermediate Goods Producers**

Firms produce with labour but they also use an *intermediate imported input* $M_{F,t}$ in their production function.

$$Y_{N,t}(i) = A_{N,t} M_{F,t}^{\kappa}(i) N_{N,t}^{1-\kappa}(i) \tag{12}$$

where $A_{N,t} = (A_{N,t}^{ss})^{1-\rho_n} A_{N,t-1}^{\rho_n} \epsilon_{N,t}$, and $\rho_n \in (0, 1]$. $M_{F,t}$ is the foreign country's final good that captures intermediate input utilisation in the production function.

Firms are monopolistically competitive and adjust prices according to Rotemberg (1982), incurring a cost each time they do so:

$$AC_t(i) = \frac{\xi}{2} \left( \frac{P_{N,t}(i)}{P_{N,t-1}(i)} - \bar{\Pi} \right)^2 Y_{N,t} P_{N,t}$$

[4] Including profits in the unconstrained agent's budget constraint or allowing for redistribution does not change our qualitative results. Our current assumption is consistent with foreigners owning the companies located in the home country and consuming all proceeds abroad.

where $\xi$ summarises the degree of nominal rigidity in the economy, and $\bar{\Pi}$ is the steady state inflation. Firms in the non-tradable sector minimise:

$$\max_{M_{F,t}(i),N_{N,t}(i),P_{N,t}(i)} P_{N,t}(i)Y_{N,t}(i) - P_{F,t}M_{F,t}(i) - W_tN_{N,t}(i) - AC_t(i)$$

subject to the technology constraint (12) and the demand function for firm's $i$ product (11). This problem yields:

$$W_t = MC_t(i)(1-\kappa)\frac{Y_{N,t}(i)}{N_{N,t}(i)} \tag{13}$$

$$P_{F,t} = MC_t(i)\kappa\frac{Y_{N,t}(i)}{M_{F,t}(i)} \tag{14}$$

where $MC_t(i)$ is the Lagrange multiplier on the technology constraint. We can interpret this multiplier as the shadow cost of producing an additional unit of good $Y_{N,t}(i)$, that is, the marginal cost. Substituting equations (13) and (14) into the production function we get the demands functions for the two inputs of production:

$$N_{N,t}(i) = \frac{Y_{N,t}(i)}{A_{N,t}}\left[\frac{1-\kappa}{\kappa}\frac{P_{F,t}}{W_t}\right]^{\kappa}, \quad M_{F,t}(i) = \frac{Y_{N,t}(i)}{A_{N,t}}\left[\frac{\kappa}{1-\kappa}\frac{W_t}{P_{F,t}}\right]^{1-\kappa}$$

The total cost function is equal to:

$$\begin{aligned} TC_t(i)\left(W_t, Y_{N,t}(i), P_{F,t}, A_{N,t}\right) &= W_tN_{N,t} + P_{F,t}M_{F,t} \\ &= W_t\frac{Y_{N,t}(i)}{A_{N,t}}\left[\frac{1-\kappa}{\kappa}\frac{P_{F,t}}{W_t}\right]^{\kappa} + P_{F,t}\frac{Y_{N,t}(i)}{A_{N,t}}\left[\frac{\kappa}{1-\kappa}\frac{W_t}{P_{F,t}}\right]^{1-\kappa} \\ &= \frac{Y_{N,t}(i)}{A_t}W_t^{1-\kappa}P_{F,t}^{\kappa}\left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right] \end{aligned} \tag{15}$$

Differentiation of the total cost function leads to the marginal cost:

$$MC_{N,t}(i) = \frac{W_t^{1-\kappa}P_{F,t}^{\kappa}}{A_t}\left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right] \tag{16}$$

Since equation (16) is solely a function of factors prices and productivity: $MC_{N,t}(i) = MC_{N,t}\ \forall i$. We can rewrite the nominal marginal cost in real terms as follows:

$$\begin{aligned} MC^r_{N,t}(i) = \frac{MC_{N,t}}{P_{N,t}} &= \frac{1}{A_t}\frac{P_t}{P_t}\frac{1}{P_{N,t}}W_t^{1-\kappa}P_{F,t}^{\kappa}\left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right] \\ &= \frac{1}{A_t}\frac{P_t}{P_{N,t}}\left(\frac{W_t}{P_t}\right)^{1-\kappa}\left(\frac{P_{F,t}}{P_t}\right)^{\kappa}\left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right] \end{aligned} \tag{17}$$

Taking the first order condition with respect to $P_{N,t}(i)$ we obtain the Phillips Curve for non-tradable goods:

$$\Pi_{N,t}\left(\Pi_{N,t} - \bar{\Pi}\right) = \beta\mathbb{E}_t\left[\Lambda^U_{t,t+1}\Pi_{N,t+1}\left(\Pi_{N,t+1} - \bar{\Pi}\right)\frac{Y_{N,t+1}}{Y_{N,t}}\right] + \frac{\epsilon}{\xi}\left(MC_t - \frac{\epsilon-1}{\epsilon}\right) \tag{18}$$

where $\bar{\pi}$ is the steady state level of inflation. Using the demand relation and the labour market clearing condition $N_{N,t} = \int_0^1 N_{N,t}(i)di$, we can write the aggregate production function as:

$$Y_{N,t}\Delta_t = A_{N,t}M_{F,t}^{\kappa}N_{N,t}^{1-\kappa}$$

where $\Delta_t = \left(1 - \frac{\xi}{2}(\Pi_N - \bar{\Pi})^2\right)$ is the index of price dispersion.

### 2.2.2 Tradable Sector

The tradable sector is internationally competitive, therefore Home firms in this sector take the prices as given at $P_{T,t} = \mathcal{E}_t P_{T,t}^*$. We assume that the dynamics of the international price $P_{T,t}^*$ are driven by developments in world markets and are thus taken as an exogenous variable by the small open economy. The tradable technology is given by:

$$Y_{T,t} = A_{T,t}N_{T,t}^{1-\zeta} \tag{19}$$

Therefore, the problem of the tradable firm is:

$$\max_{N_{T,t}} P_{T,t}Y_{T,t} - W_t N_{T,t}$$

subject to technological constraints. This leads to:

$$\frac{W_t}{P_{T,t}} = (1-\zeta)A_{T,t}N_{T,t}^{-\zeta} \implies W_t N_{T,t} = (1-\zeta)Y_{T,t}P_{T,t} \tag{20}$$

which gives us profits $\Psi_{T,t} = (\zeta)P_{T,t}Y_{T,t}$. Finally, similarly to $P_{F,t}^*$, the foreign price index for the tradable sector is given by:

$$P_{T,t}^* = (P_{T,ss}^*)^{1-\rho_T}(P_{T,t-1}^*)^{\rho_T}\epsilon_{T,t} \tag{21}$$

with $\rho_T \in (0,1]$.

## 2.3 Monetary Policy

The monetary authority sets the interest rate according to the following Taylor rule:

$$\frac{R_t}{R_{ss}} = \left(\frac{\Pi}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y}{\bar{Y}}\right)^{\phi_y}\exp(\nu_m) \tag{22}$$

where $\nu_m \sim N(0, \sigma_m^2)$ is a monetary policy shock, which follows an AR(1) process, and $\bar{Y}$ is the steady state level of output.

## 2.4 Equilibrium

Given the tradable and foreign imported prices $P^*_{T,t}$, $P^*_{F,t}$, the monetary policy rule determining $i_t$, foreign output, inflation and interest rates $Y^*_t, \Pi^*_t, i^*_t$, and an initial condition on price dispersion, equilibrium in the economy is given by a sequence of quantities $\{C_{H,t}, C_{T,t}, C_{N,t}, C_{F,t}, C_t, C^U_t, C^C_t, N_t, B^H_{t+1}, B^*_{t+1}, Y_{N,t}, Y_{T,t}, M_{F,t}, \Psi_t\}_{t=0}^{\infty}$ and prices $\{\Lambda_{t,t+1}, \Pi_{H,t}, \Pi_{N,t}, \Pi_{T,t}, \Pi_{F,t}, \Pi_t, W_t, \mathcal{T}_t, \mathcal{S}_t, \mathcal{E}_t, \Delta_t\}_{t=0}^{\infty}$ such that firms and households maximise their objectives, and the goods, labour and financial markets clear, we obtain:

$$\begin{aligned}
Y_t &= C_{T,t} + C^*_{T,t} + C_{N,t} \\
Y_{T,t} &= C_{T,t} + C^*_{T,t} \\
C_{N,t} &= Y_{N,t} \\
B_t &= 0 \\
N_t &= N_{T,t} + N_{N,t} = N^C_t \lambda + N^U_t (1-\lambda) \\
Y^*_t &= C^*_t
\end{aligned}$$

We need to derive the demand functions for $Y_{T,t}$and $Y_{N,t}$ to obtain the market clearing conditions. Tradables are by definition consumed both at home and in the RoW:

$$\begin{aligned}
C_{T,t} &= \gamma \left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu} C_{H,t} = \gamma \left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu} (1-\alpha) \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t = \gamma(1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu} \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t \\
C^*_{T,t} &= \gamma\alpha \left(\frac{P_t}{P_{T,t}}\right)^{\eta} \mathcal{S}_t C^*_t \\
Y_{T,t} &= \gamma(1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu} \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t + \gamma\alpha \left(\frac{P_t}{P_{T,t}}\right)^{\eta} \mathcal{S}_t C^*_t
\end{aligned}$$

where $C^*_{T,t}$, the foreign tradable demand, is derived by assuming symmetric preferences in the rest of the world. For non-tradables, instead, we calculate domestic demand as follows:

$$\begin{aligned}
C_{N,t} &= (1-\gamma) \left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu} C_{H,t} \\
C_{N,t} &= (1-\gamma) \left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu} (1-\alpha) \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t \\
C_{N,t} &= (1-\alpha)(1-\gamma) \left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu} \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t
\end{aligned}$$

Therefore,

$$Y_t = (1-\alpha)(1-\gamma) \left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu} \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t + \gamma(1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu} \left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t + \gamma\alpha \left(\frac{P_t}{P_{T,t}}\right)^{\eta} \mathcal{S}_t C^*_t$$

### 2.4.1 Natural Level of Output and Interest Rate

To understand the nature of our shocks and where the real policy interest rate should be headed, we need to calculate the natural level of output $Y^n_t$ and natural real interest rate $r^n_t$.

$Y_t^n$ is the level of output that would arise under flexible prices. To derive it, we need to determine the profit-maximizing flexible price for the domestic non-tradable good firms - the only sector facing nominal rigidities. Profit-maximising firms set the flexible optimal price so as to equalise the marginal cost to the marginal revenue. This is equivalent to setting the real marginal cost to the inverse of the desired markup:

$$MC_{N,t} = \frac{\epsilon - 1}{\epsilon}$$

Using the real marginal cost as per equation (17), this leads to:

$$\frac{\epsilon - 1}{\epsilon} = \frac{1}{A_{N,t}} \frac{P_t}{P_{N,t}} \left(\frac{W_t}{P_t}\right)^{1-\kappa} \left(\frac{P_{F,t}}{P_t}\right)^{\kappa} \left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right] \tag{23}$$

Assuming, without loss of generality, that in steady state $C^U = C^C = C$, we can write the labour supply condition as follows:[5]

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\phi}$$

Therefore:

$$\frac{\epsilon - 1}{\epsilon} = \frac{1}{A_{N,t}} \frac{P_t}{P_{N,t}} \left(C_t^{\sigma} N_t^{\phi}\right)^{1-\kappa} \left(\frac{P_{F,t}}{P_t}\right)^{\kappa} \left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa} + \left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right] \tag{24}$$

We next express $C_t$ in terms of $C_t^*$. Using the first-order condition for consumption from the household problem, and assuming that the same conditions hold abroad:

$$\frac{1}{(1+i_t)} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{(1+\pi_{t+1})}\right] \tag{25}$$

$$\frac{1}{(1+i_t^*)} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{1}{(1+\pi_{t+1}^*)}\right] \tag{26}$$

Given that in steady state $\pi = \pi^* = 1$, then $r = r^* = \frac{1}{\beta}$. We can hence write:

$$[1 + \chi(b_t^* - \bar{b}^*)] = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{C_t^*}{C_{t+1}^*}\right)^{-\sigma} \frac{1}{\beta} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}\right]$$

$$[1 + \chi(b_t^* - \bar{b}^*)] = \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{C_t^*}{C_{t+1}^*}\right)^{-\sigma} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}\right]$$

$$[1 + \chi(b_t^* - \bar{b}^*)] = \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{C_{t+1}^*}{C_t^*}\right)^{\sigma} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}\right]$$

$$\mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\right] = \mathbb{E}_t \left[\left(\frac{C_t^*}{C_{t+1}^*}\right)^{\sigma} \frac{\mathcal{S}_t}{\mathcal{S}_{t+1}}\right] [1 + \chi(b_t^* - \bar{b}^*)], \tag{27}$$

[5] Galí, López-Salido, and Vallés (2007).

which is the resulting international risk-sharing condition. Notice that in steady state, we are back to the condition that would arise in a perfect risk-sharing scenario with symmetric countries. Moreover,

$$
\begin{aligned}
\mathcal{D}_t^{\frac{1}{\sigma}} &= \frac{C_t}{C_t^* \mathcal{S}_t} \qquad \mathcal{D}_t^{\frac{1}{\sigma}} = (1+\chi(b_t^* - \bar{b})) \neq 1, \quad D = 1 \\
C_t &= \mathcal{D}_t^{\frac{1}{\sigma}} C_t^* \mathcal{S}_t \\
\frac{C_t}{\mathcal{D}_t^{\frac{1}{\sigma}}} &= C_t^* \mathcal{S}_t
\end{aligned}
$$

Using the good market clearing equation, and the assumption that international markets are incomplete, we can write:

$$
\begin{aligned}
Y_t &= (1-\alpha)(1-\gamma)\left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t + \gamma(1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta} C_t + \gamma\alpha\left(\frac{P_t}{P_{T,t}}\right)^{\eta} \mathcal{S}_t C_t^* \\
Y_t &= \left[(1-\alpha)(1-\gamma)\left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta} + \gamma(1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta} + \frac{\gamma\alpha}{\mathcal{D}_t^{\frac{1}{\sigma}}}\left(\frac{P_t}{P_{T,t}}\right)^{\eta}\right] C_t \\
C_t &= \frac{Y_t}{\Sigma_{\alpha\gamma,t}}
\end{aligned}
$$

where $\Sigma_{\alpha\gamma,t} = \left[(1-\alpha)(1-\gamma)\left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta} + \gamma(1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta} + \frac{\gamma\alpha}{\mathcal{D}_t^{\frac{1}{\sigma}}}\left(\frac{P_t}{P_{T,t}}\right)^{\eta}\right]$. $\Sigma_{\alpha,\gamma,t}$ describes the relationship between domestic consumption and aggregate domestic production. In an open economy, these two variables do not move in lockstep; instead, their dynamics are influenced by the degree of openness to trade. A higher $\alpha$ widens the wedge between consumption and production, attributable to the increased share of tradable goods destined for export.

In the absence of distortions, we can obtain $N_t$ by aggregating labour in each sector:

$$
N_{T,t} = \left(\frac{Y_{T,t}}{A_{T,t}}\right)^{\frac{1}{1-\zeta}} \tag{28}
$$

$$
N_{N,t} = Y_{N,t}\left(\frac{P_{F,t}}{P_{N,t}}\right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \tag{29}
$$

Since $N_t = N_{N,t} + N_{T,t}$, we can write:

$$
\begin{aligned}
N_t &= \left(\frac{Y_{T,t}}{A_{T,t}}\right)^{\frac{1}{1-\zeta}} + Y_{N,t}\left(\frac{P_{F,t}}{P_{N,t}}\right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}} \\
&= A_{T,t}^{-\frac{1}{1-\zeta}}\left(\left(\gamma(1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\mathcal{D}_t^{\frac{1}{\sigma}} + \gamma\alpha\left(\frac{P_t}{P_{T,t}}\right)^{\eta}\right)\mathcal{S}_t C_t^*\right)^{\frac{1}{1-\zeta}} \\
&+ (1-\alpha)(1-\gamma)\left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\mathcal{D}_t^{\frac{1}{\sigma}}\mathcal{S}_t C_t^*\left(\frac{p_{F,t}}{p_{N,t}}\right)^{\frac{1-\kappa}{\kappa}} \kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}}
\end{aligned}
$$

Substituting back in equation (24), we obtain:

$$\frac{\epsilon-1}{\epsilon}\frac{A_{N,t}p_{N,t}}{\Gamma}p_{F,t}^{-\kappa}=\left(C_t^{\sigma}N_t^{\phi}\right)^{1-\kappa}$$
$$C_t^{\sigma}N_t^{\phi}=\left(\frac{\epsilon-1}{\epsilon}\frac{A_{N,t}p_{N,t}}{\Gamma}p_{F,t}^{-\kappa}\right)^{\frac{1}{1-\kappa}}$$
$$Y_t^n=\Sigma_{\alpha\gamma,t}\left[\left(\frac{\epsilon-1}{\epsilon}\frac{A_{N,t}p_{N,t}}{\Gamma}p_{F,t}^{-\kappa}\right)^{\frac{1}{1-\kappa}}N_t^{-\phi}\right]^{\frac{1}{\sigma}}$$

where $\Gamma=\left[\left(\frac{\kappa}{(1-\kappa)}\right)^{1-\kappa}+\left(\frac{(1-\kappa)}{\kappa}\right)^{\kappa}\right]$. Substituting $N_t, Y_t^*=C_t^*$,

$$Y_t^n=\Sigma_{\alpha\gamma,t}\left[\left(\frac{\epsilon-1}{\epsilon}\frac{A_{N,t}p_{N,t}}{\Gamma}p_{F,t}^{-\kappa}\right)^{\frac{1}{1-\kappa}}\right.$$
$$\left(A_{T,t}^{-\frac{1}{1-\zeta}}\left(\left(\gamma(1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\mathcal{D}_t^{\frac{1}{\sigma}}+\gamma\alpha\left(\frac{P_t}{P_{T,t}}\right)^{\eta}\right)\right)\mathcal{S}_tC_t^*\right)^{\frac{1}{1-\zeta}}$$
$$\left.+(1-\alpha)(1-\gamma)\left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\mathcal{D}_t^{\frac{1}{\sigma}}\mathcal{S}_tC_t^*\left(\frac{p_{F,t}}{p_{N,t}}\right)^{\frac{1-\kappa}{\kappa}}\kappa^{-\frac{\kappa}{1-\kappa}}A_{N,t}^{-\frac{1}{1-\kappa}}\right)^{-\phi}\right]^{\frac{1}{\sigma}}$$

$$\begin{aligned}Y_t^n=\Sigma_{\alpha\gamma,t}&\left(\mu^{\frac{1}{1-\kappa}}A_{N,t}^{\frac{1}{1-\kappa}}p_{N,t}^{\frac{1}{1-\kappa}}\Gamma^{-\frac{1}{1-\kappa}}p_{F,t}^{-\frac{\kappa}{1-\kappa}}\right)^{\frac{1}{\sigma}}\\&\left(A_{T,t}^{-\frac{1}{1-\zeta}}\left(\left(\gamma(1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\mathcal{D}_t^{\frac{1}{\sigma}}+\gamma\alpha\left(\frac{P_t}{P_{T,t}}\right)^{\eta}\right)\right)\mathcal{S}_tC_t^*\right)^{\frac{1}{1-\zeta}}\\&\left.+(1-\alpha)(1-\gamma)\left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\mathcal{D}_t^{\frac{1}{\sigma}}\mathcal{S}_tC_t^*\left(\frac{p_{F,t}}{p_{N,t}}\right)^{\frac{1-\kappa}{\kappa}}\kappa^{-\frac{\kappa}{1-\kappa}}A_{N,t}^{-\frac{1}{1-\kappa}}\right)^{-\phi}\right]^{\frac{1}{\sigma}}\end{aligned}\tag{30}$$

$\mu$ is the gross markup in steady state. Equation (30) describes the natural output, which can vary with technology in both sectors, relative prices of input and foreign demand. Finally, the natural real interest rate is the risk-free real interest rate consistent with the Euler equation when output is at its natural level at all times:

$$(C_{t+1}^n)^{\sigma}=\beta(1+r_t^n)(C_t^n)^{\sigma}$$
$$(1+r_t^n)=\frac{1}{\beta}\left(\left(\frac{Y_{t+1}^n}{Y_t^n}\right)^{\sigma}\frac{\Sigma_{\alpha\gamma,t}}{\Sigma_{\alpha\gamma,t+1}}\right)$$

# 3 Calibration

We calibrate the model at a quarterly frequency. Our aggregate baseline calibration is standard, as several preferences and technology parameters are shared with the standard New

Keynesian literature. For our baseline scenario, we set the share of hand-to-mouth consumers in the population equal to 30% (Kaplan, Violante, and Weidner (2014), Kaplan, Moll, and Violante (2018), Kaplan and Violante (2022)). To allow for a more realistic representation of the households' optimisation problem, we set the elasticity of intertemporal substitution $\sigma = 4$.[6] This aligns with recent literature (Jones (2023), Kimball, Reck, Zhang, Ohtake, and Tsutsui (2024), etc.) that discusses the limitations of log-utility assumptions in economics models. For simplicity, we consider standard unit values for the elasticity of substitution between foreign and domestic goods $\eta$, the elasticity of substitution between tradable and non-tradable goods $\nu$, and the Frisch elasticity.[7] In the baseline model, $\alpha$ equals 0.25, which

| Parameter | Benchmark Model | Parameter | Benchmark Model |
|---|---|---|---|
| $\beta$ | 0.99 | $\chi$ | 0.00001 |
| $\alpha$ | 0.25 | $\phi_\pi$ | 2 |
| $\epsilon$ | 6 | $\phi_y$ | 0 |
| $\eta$ | 1 | $\rho_s$ | 0.9 |
| $\nu$ | 1 | $\xi$ | 28.003 |
| $\lambda$ | 0.3 | $\zeta$ | 0.7 |
| $\gamma$ | 0.2 | $\phi$ | 1 |
| $\sigma$ | 4 | $\kappa$ | $\approx 0$ |

Table 1: *Notes:* $\rho_s$ where $s \in \{T, a_N, a_T, i, y^*\}$.

implies a home bias equal to 0.75, following Harrison and Oomen (2010). We set $\kappa$, the income share of foreign primitive input in the production of non-tradable goods, to 0 in the baseline. However a positive $\kappa$ (around 0.3 to match the labour share of income) exacerbates quantitatively the effect of fragmentation scenarios we study, without changing the qualitative interpretation. Finally, we choose $\gamma$, the share of tradable goods in the consumption basket to be 0.2.

## 3.1 Special Case: RANK

To establish basic intuition for our results, we conduct our three main exercises in a representative agent model. This corresponds to a special case of our baseline model, where the share of constrained households is $\lambda = 0$ (implying a heterogeneity index of $\Gamma_t = 1$).

**Gradual Fragmentation** To simulate a gradual shift towards a more restricted trade environment, Figure 4 plots the impulse response functions (IRFs) of various macroeconomic aggregates to a gradual increase in import prices ($P_{F,t}$). In this scenario, import prices stabilise

[6] The results are robust for other values of $\sigma \neq 1$.

[7] However, results are robust to usual variation in these parameters.

in the medium term, with a cumulative increase of 100 percent. The price of imported goods, $P_{F,t}$, will affect demand directly, both through the consumption basket $C_{F,t}$ and through imported inputs $M_{F,t}$ in the production of non-tradables. Additionally, it indirectly affects demand through real wages.

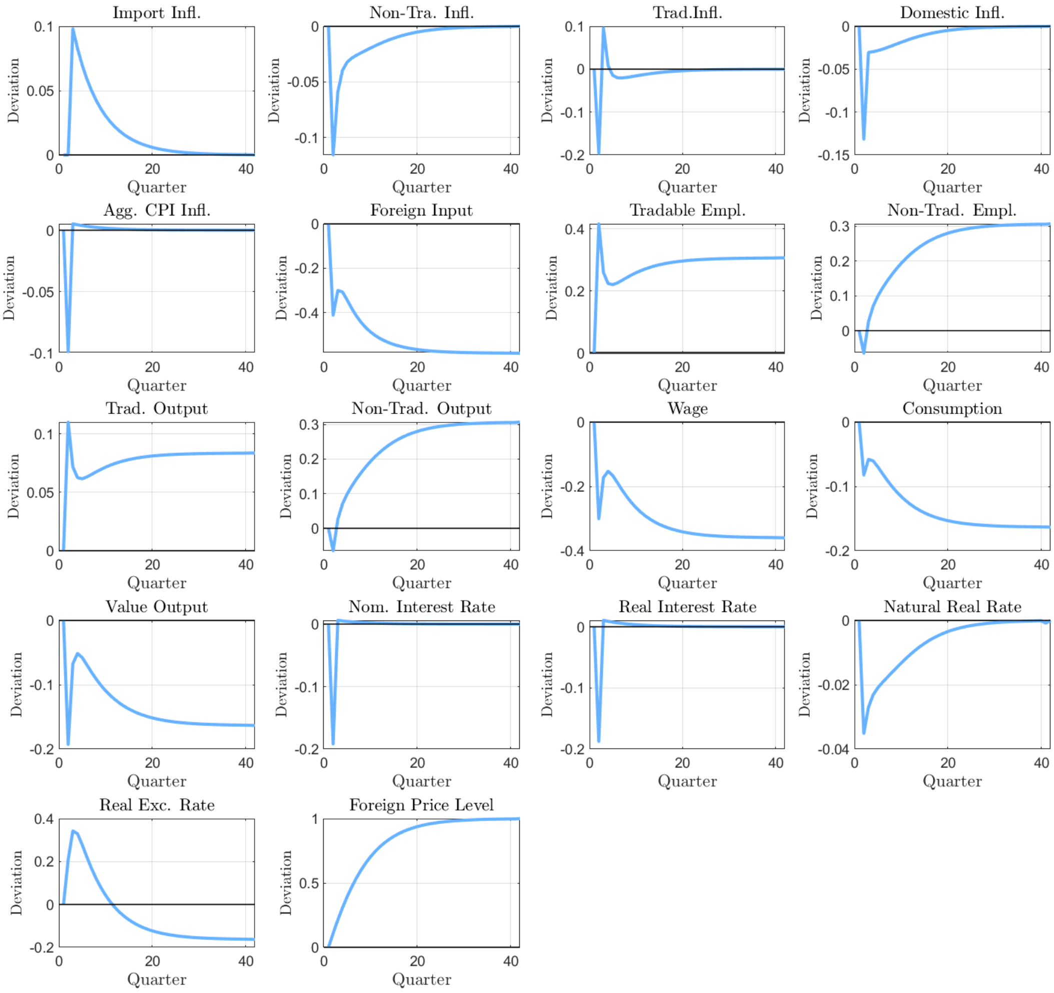
As the price of foreign goods gradually increases, there is an increase in imported inflation. This places upward pressure on CPI inflation, but is more than offset by the fall in domestic inflation, which declines due to reductions in both non-tradable and tradable inflation. This fall in domestic inflation is driven by the fall in consumption, which responds to the fall in permanent real income. Households partly compensate for the fall in real wages by increasing labour supply, which mitigates the impact of higher import prices on aggregate supply. Overall, the anticipation effect of lower real incomes leads to demand falling more than supply, and consequently, to a fall in domestic inflation. CPI inflation, which is a composite of domestic and imported good inflation, falls on balance. This is reflected in a decrease in the natural real rate of interest, indicating that when demand materially adjusts in anticipation, the effect can be disinflationary. This prompts the central bank to ease policy, by lowering the nominal interest rate, in line with the rule characterising its reaction function.

**Front-loaded Fragmentation** Figure 5 shows the effect of a permanent and immediate increase in foreign prices $P_{F,t}$. This type of shock is intended to capture rapid 'fragmentation events' like price or – indirectly – tariff increases.[8]

The increase in the price of the foreign good leads to a sharp increase in imported inflation, which quickly reverts back to the steady-state level. Aggregate consumption falls on impact, and following some short-term volatility reflecting interest rates and employment, stabilises at a lower steady state. Non-tradable inflation falls at first, reflecting the fall in consumption. As non-tradable firms gradually pass on higher marginal costs, non-tradable inflation increases temporarily. Tradable inflation also falls initially, following the fall in consumption. Domestic inflation rises sharply, driven by the surge in tradables inflation. Combined with imported inflation, this leads to a significant spike in aggregate CPI inflation. Non-tradable output falls temporarily as the higher price of the foreign input restricts supply, but then recovers as households increase labour supply to compensate for their income losses. The economy enters a temporary period of tradeoff or stagflation, with prices increasing and consumption decreasing. Overall, the natural rate decreases on impact and then slightly increases before going back to the steady state. The Taylor rule followed by the

[8] A caveat is in order: we do not consider the uses of fiscal proceeds from the tariffs in the analysis. One way to justify this would be to assume that proceeds from tariffs are used to stimulate supply and demand in equal amounts, without affecting the output gap or inflation. Alternatively, we can assume that import restrictions take the form of non-tariff barriers (which comprise the majority of trade restrictions), in which case there is no tax revenue to be rebated. More broadly, this exercise is intended to capture the realignment of trade, whereby geopolitics forces domestic firms to switch from low-cost to geopolitically friendly suppliers, leading to efficiency losses.

Figure 4: IRFs to a Gradual Increase in Foreign Price Level.

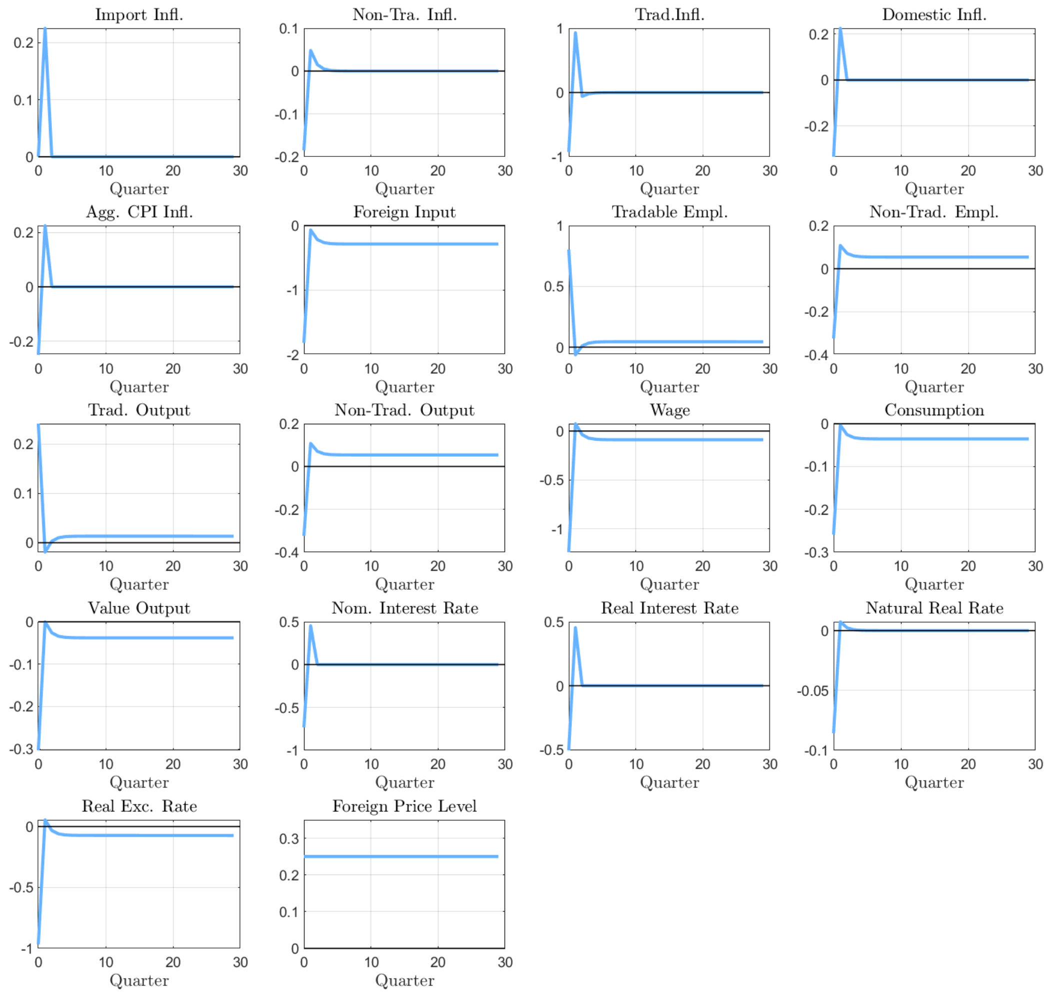



*Notes:* IRFs to a 100% gradual positive foreign price shock. The results are generated under a RANK calibration ($\lambda = 0$). All the other parameters are calibrated according to Table 1.

central bank, along with the volatility in CPI inflation, leads to volatility in the nominal rate, which eventually increases to return inflation to target. On the external side of the economy, the real exchange rate reflects the nature of the shock and the policy response, depreciating on impact and then appreciating as the interest rate tightens.

**Fall in Tradables Productivity** $A_{T,t}$ We consider an additional shock that can result from "trade fragmentation": a persistent decrease in the productivity of the tradable goods sector, which makes the Home production of tradable goods less competitive in the global market. In Figure 6, we show the responses of all variables to a negative one standard deviation

Figure 5: IRFs to a Front-loaded Increase in Foreign Price Level.

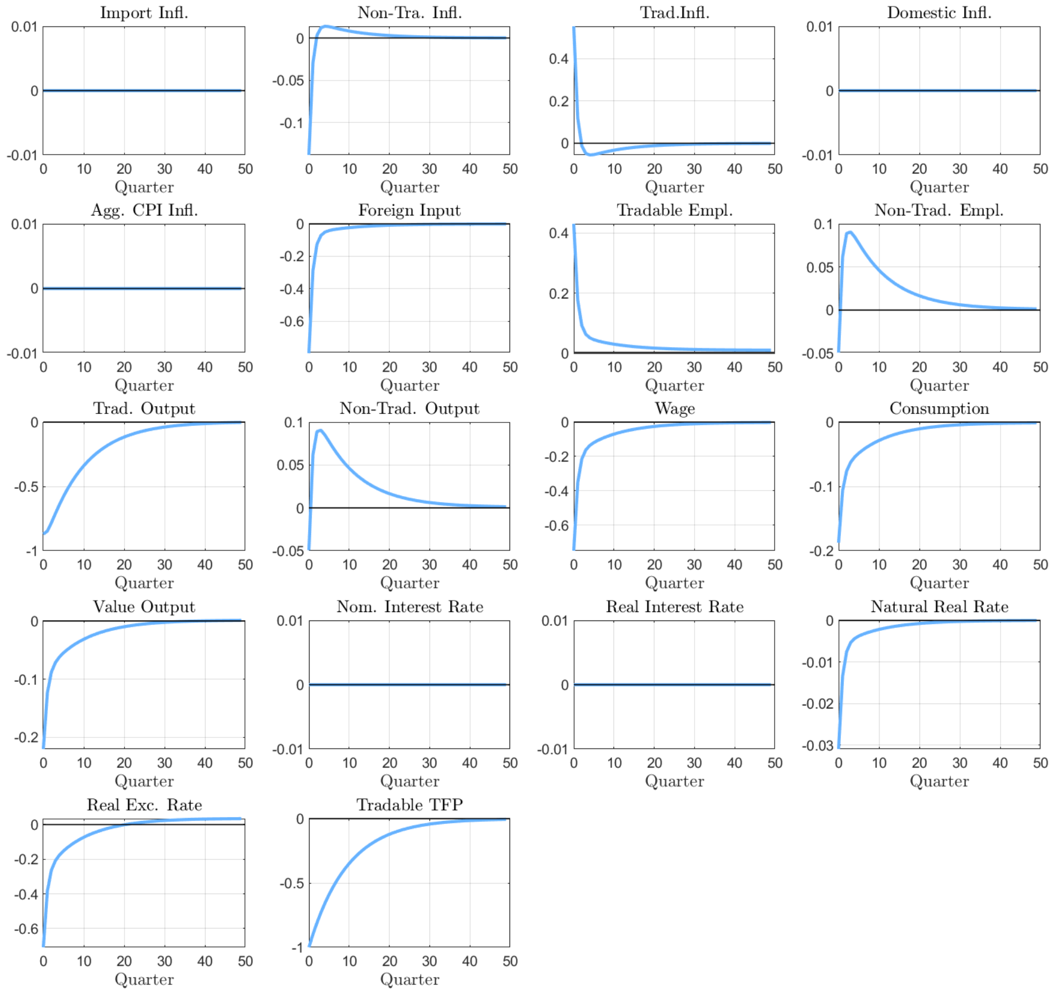



*Notes:* IRFs to a 25% positive foreign price shock. The results are generated under a RANK calibration ($\lambda = 0$). All the other parameters are calibrated according to Table 1.

shock deviation of total factor productivity in the tradable sector, $A_{T,t}$ (equivalent to 100 percentage-point deviation from the steady state). The shock is persistent but not permanent, as it reverts back to steady state in the medium-to-long term.

As in the standard New Keynesian model, this constraint on supply results in a fall in tradable output and an increase in tradable goods inflation as marginal costs rise. The fall in tradable output leads to a decrease in labour demand in this sector. Households cut consumption in response to the negative income effect of lower incomes. They also increase their labour supply to make up for some of the losses. From the perspective of tradable goods firms, the fall in nominal wages reduces the cost of producing a given level of output, offsetting the loss of productivity. The increase in household labour supply is, however, not enough to counteract the negative income effect. The fall in consumption also leads to a decrease in demand for non-tradable goods and inflation. Domestic and aggregate CPI inflation remain unchanged as the fall in non-tradable goods prices offset the increase in tradable goods prices. Since monetary policy reacts only to changes in inflation, real and nominal rates do not move. On impact, the natural rate decreases temporarily, reflecting lower demand, but it increases moderately over time as the economy recovers.

Figure 6: IRFs to a Negative Shock to Tradable TFP.




*Notes:* IRFs to a negative TFP shock in the tradable sector. The results are generated under a RANK calibration ($\lambda = 0$). All the other parameters are calibrated according to Table 1.

To summarise, since all three scenarios constrain supply capacity, the supply side effects are unambiguous. Marginal costs increase initially and are passed through only gradually to prices due to price rigidities. In partial equilibrium, as nominal expenditures are fixed, this leads to a fall in household demand for goods from the unaffected sector. To capture general equilibrium effects, this section uses a simple framework to demonstrate how aggregate demand adjusts differently in various scenarios that model aspects of trade fragmentation. In the gradual fragmentation scenario, a steady increase in import prices reduces the purchasing power of labour income, through an increase in the price of imported consumption goods as well as through a fall in nominal wages. If this change is expected to be permanent, then households also expect a permanent fall in purchasing power, leading to a fall in consumption spending. Therefore, a fall in permanent labour income leads to a fall in demand, which affects the price level response to the initial increase in import prices. This scenario leads to stagnation, with lower real incomes and low inflationary pressures. In contrast, a front-loaded fragmentation scenario (sharp permanent increase in import prices) may create a short-term tradeoff for policymakers. Finally, persistent falls in tradable sector productivity might end up being neutral for inflation.

These results suggest that the form in which fragmentation materialises, the extent to which it is anticipated by households, and households' ability to smooth consumption over time, all matter. The next section will consider the case where a proportion of households are unable to smooth consumption in response to changes in their permanent income.

## 3.2 TANK Case

An important factor in gauging how inflation will respond to these trade-related shocks is the degree of forward-looking behaviour in demand. Specifically, it hinges on the extent to which agents can effectively smooth consumption in the presence of a shock. Therefore, this section considers a more general framework that allows for household heterogeneity.

Relative to the previous section, the presence of constrained households introduces agents who cannot smooth consumption in response to the shock, although they can adjust their labour supply. The fall in real incomes affects these households directly, while the fall in labour demand and aggregate demand by unconstrained households affects them indirectly. As in the previous section, we consider three scenarios to show how the form of fragmentation will affect the demand-side adjustment.

We also explore how much the extensive margin of household heterogeneity matters by considering two cases: an economy in which roughly one-third of agents are constrained (hand-to-mouth) ($\lambda = 0.3$) compared with an economy where roughly three-quarters of agents are constrained (hand-to-mouth) ($\lambda = 0.8$). We find that for this degree of variation, heterogeneity does not alter the main results. Financially unconstrained consumers, even in small shares, can trigger a fall in demand, which spills over to the financially constrained agents.[9]

The fall in permanent income highlighted in the previous section will be mitigated by the proportion of constrained households. While all households consume out of permanent income in a RANK model, only a proportion $\lambda$ of households do so in a TANK model. The presence of constrained households lessens the adverse demand side effect since they cannot cut consumption in anticipation of the shock.[10]

**Gradual Fragmentation** To simulate a gradual shift towards a more restricted trade environment, Figure 7 plots the impulse response functions to an increase in the price of imported goods, which stabilises at a 100 percent higher level in the medium term. As in the RANK case, the price of imported goods, $P_{F,t}$, affects demand directly, both through the consumption basket $C_{F,t}$ and through imported inputs $M_{F,t}$ in production. Additionally, it indirectly affects demand through real wages.

The response of real and nominal variables with a higher share of constrained agents is similar to the RANK case (Figure 4). However, the response of both households' consumption when $\lambda = 0.8$ differs. In particular, constrained households' consumption falls slightly more over time, while unconstrained households are able to borrow to smooth consumption. Unconstrained household consumption increases initially, but then falls below steady state and never recovers. In the case of $\lambda = 0.8$, fewer unconstrained consumers anticipate the permanent income losses and adjust consumption in response, mitigating the fall in aggregate demand.

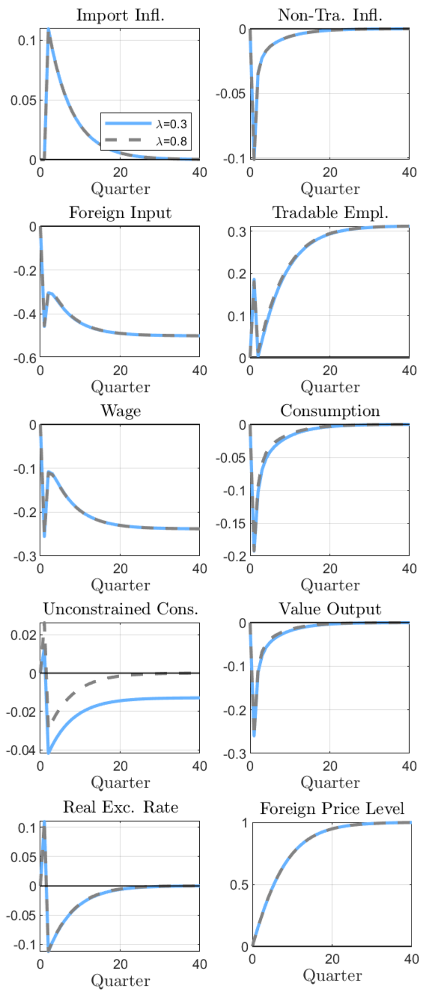
As unconstrained households cut consumption in response to the fall in their permanent real income, demand spillovers lead to lower labour demand and lower wages for constrained households. In addition, constrained households cut consumption due to a fall in their real disposable income, which decreases due to higher prices. Constrained households partly compensate for the loss in real wages by increasing labour supply, which mitigates the impact of higher import prices on aggregate supply. Overall, the anticipation effect of lower real incomes on demand by unconstrained households leads to demand falling more than supply, and consequently, to a fall in domestic inflation. The fall in domestic inflation

[9] The standard transfer rule assumed in the literature is not innocuous. As a technical assumption intended to simplify the steady state solution, this redistribution mutes the effect of household heterogeneity in our model. We therefore neutralise the effects of transfers by assuming profits are "lost at sea".

[10] The rise in import prices could potentially affect aggregate demand through distributional effects if the two households differ in their sources of income. The contractionary demand-side effect of fragmentation may be larger if labour income falls and profit income increases. Elasticities of substitution between production inputs determine the magnitude of the demand-side effect of energy price shocks in Auclert, Monnery, Rognlie, and Straub (2023) and Chan, Diz, and Kanngiesser (2024), while trade elasticities affect the real income channel of exchange rate depreciations in Auclert, Rognlie, Souchier, and Straub (2021). We abstract from these effects here in assuming unitary elasticity of substitution between inputs in nontradables production and the same between home and foreign goods in consumption (in addition to profits "lost at sea"). This is consistent with the view that trade fragmentation entails a shift from low-cost suppliers to friendly suppliers, or that elasticities of substitution are high in the medium to long run (if fragmentation is a permanent, phased-in process).

more than offsets the persistent increase in imported inflation, leading to a fall in aggregate CPI. The natural real rate of interest falls, suggesting that when demand adjusts in anticipation, the effect can be disinflationary. The nominal interest rate falls in response, in line with the rule characterising its reaction function.

Figure 7: IRFs to a Gradual Increase in Foreign Price Level.




*Notes:* $\lambda = 0.3$ in blue, $\lambda = 0.8$ in grey dashed line. All the other parameters are calibrated according to Table 1.

**Front-loaded Fragmentation** Figure 8 shows the effect of a permanent and immediate increase in foreign prices $P_{F,t}$. The responses of real and nominal variables with a higher share of constrained agents ($\lambda = 0.8$, dashed grey lines) are similar to the RANK case (Figure 5). The immediate impact of the shock suggests that the share of constrained households matters less, as the trough in unconstrained consumption is only slightly higher in the case of $\lambda = 0.8$. For this range of variation in the share of hand-to-mouth consumers, there is no significant difference in the transmission of the import price shock across the two economies.

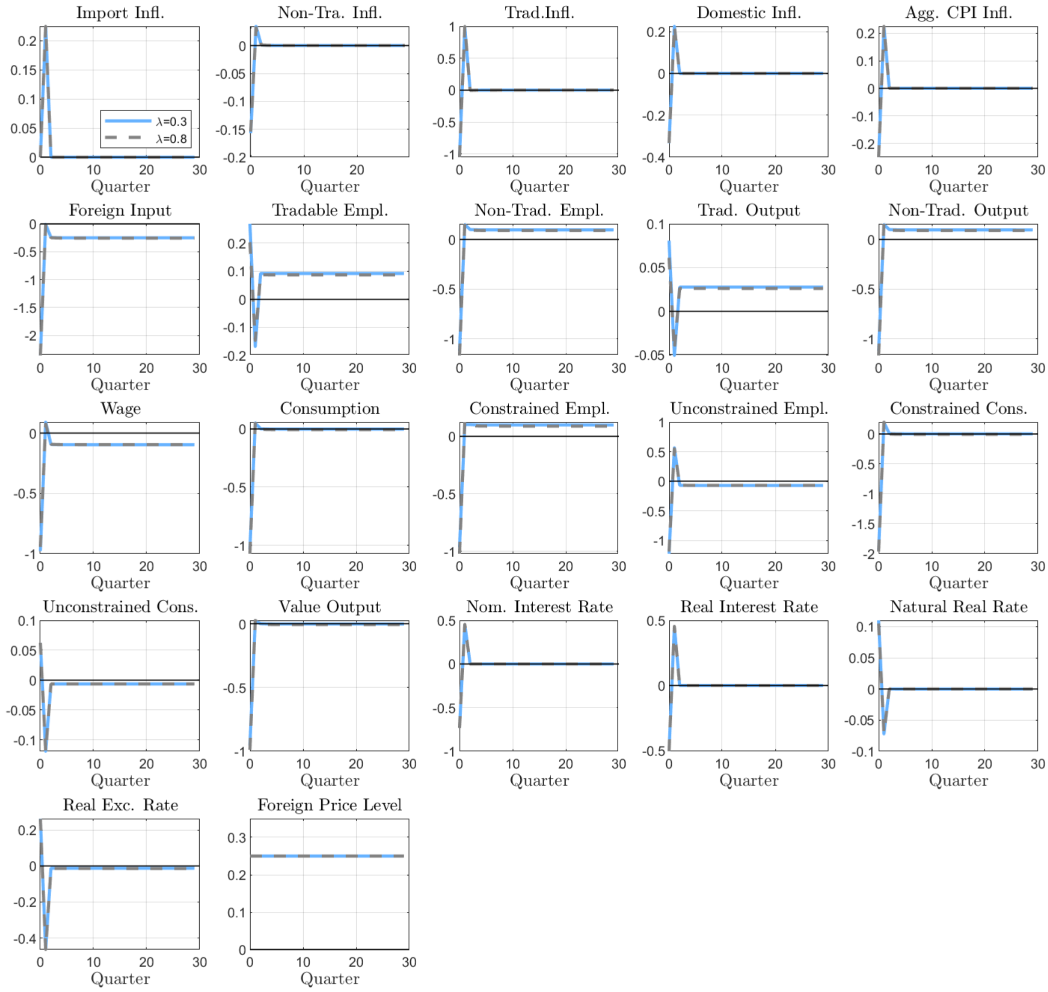
This scenario leads to a sharp increase in imported inflation, which quickly reverts back to the steady-state level. This frontloaded price shock is immediately transmitted to the constrained consumers as they are unable to smooth. The initial sharp fall in consumption is driven by the constrained households who react to the fall in purchasing power by increasing labour supply. However, as constrained households respond by increasing labour supply to make up for lower incomes, constrained consumption does not fall as much and it recovers to the new steady state level. On impact, consumption goes below steady state, before settling at a lower steady state level for both constrained and unconstrained consumers. Aggregate consumption falls on impact and stabilises at a lower steady state value. Tradable and non-tradable output fall temporarily as the higher price of the foreign input restricts supply, but then recover as households increase their labour supply to compensate for their income losses. Aggregate and domestic inflation increase and the economy enters a temporary period of tradeoff or stagflation. The monetary policy response to inflation volatility leads to volatility in the nominal rate, which eventually increases to return inflation to target. Overall, the natural rate increases on impact and then falls. Due to the nature of the shock and the policy response, the real exchange rate depreciates on impact and then appreciates as policy tightens.

**Fall in Tradables Productivity $A_{T,t}$.** Finally, we consider a persistent decrease in the productivity of the tradable goods sector, which makes the Home production of tradable goods less competitive in the global market. Figure 9 plots the responses of all variables to a one standard deviation negative shock to total factor productivity in the tradable sector, $A_{T,t}$. The shock is persistent but not permanent, as it reverts back to steady state in the medium-to-long term.

As in the standard New Keynesian model, tradable output decreases, while tradable inflation increases. This is due to the rise in the marginal cost of production caused by the fall in productivity. Consumption decreases, almost entirely driven by the constrained agents. Financially unconstrained consumers also adjust their consumption downwards, but because they can smooth over infinite lives and the shock is not permanent, the adjustment in consumption is very small. The adjustment in consumption also reflects a negative income effect coming from lower wages, as constrained consumers increase their labour supply in response to the losses on consumption.

For tradable goods firms, higher labour supply can partially compensate for the loss of productivity. As a result, labour demand in this sector increases, leading to an increase in tradable employment. While constrained agents substantially increase their labour supply, this is not enough to counteract the negative income effect. The drop in consumption also

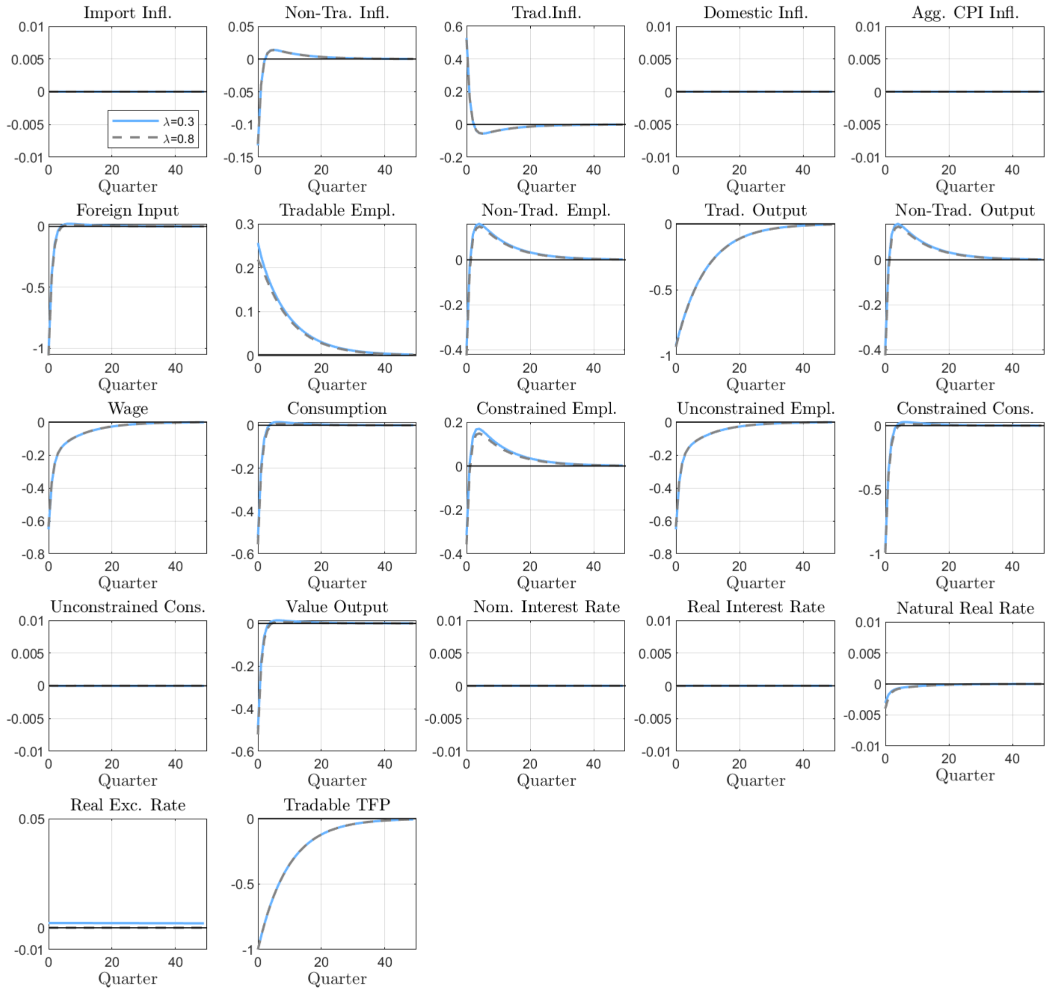
Figure 8: IRFs to a Front-loaded Increase in Foreign Price Level.




*Notes:* $\lambda = 0.3$ in blue, $\lambda = 0.8$ in grey dashed line. All the other parameters are calibrated according to Table 1.

temporarily affects non-tradable demand and inflation adversely. Tradable output recovers over time as productivity improves. Inflation falls in the non-tradable sector to offset the increase in inflation in the tradable sector. Consequently, domestic and aggregate inflation remains unchanged. Since the central bank reacts only to changes in inflation, real and nominal rates do not move. On impact, the natural rate decreases temporarily, reflecting lower demand, but it increases moderately as the economy recovers.

Figure 9: IRFs to a Negative Shock to Tradable TFP.




*Notes:* $\lambda = 0.3$ in blue, $\lambda = 0.8$ in grey dashed line. All the other parameters are calibrated according to Table 1.

In summary, the form in which fragmentation takes place will matter for the balance of supply and demand. The sudden implementation of tariffs will differ from a gradual implementation, and from an adverse shock to tradables productivity. The presence of fewer forward-looking agents mitigates the fall in aggregate demand, and more so in the scenario where the permanent increase in foreign prices takes place gradually.

### 3.3 Different Degrees of Openness

We explore the role of the initial level of openness to trade in the macroeconomic reaction to fragmentation. We hence study the same set of fragmentation scenarios while varying the level of home bias, denoted by $\alpha$. This adjustment allows us to capture how different economies experience trade shocks with varying degrees of intensity, and how policy responses may differ depending on the level of openness of each country. A lower value of $\alpha$ indicates that the consumption basket and price index are *less* influenced by foreign prices and goods and the economy is less open. Consequently, consumption relies more heavily on domestic production.

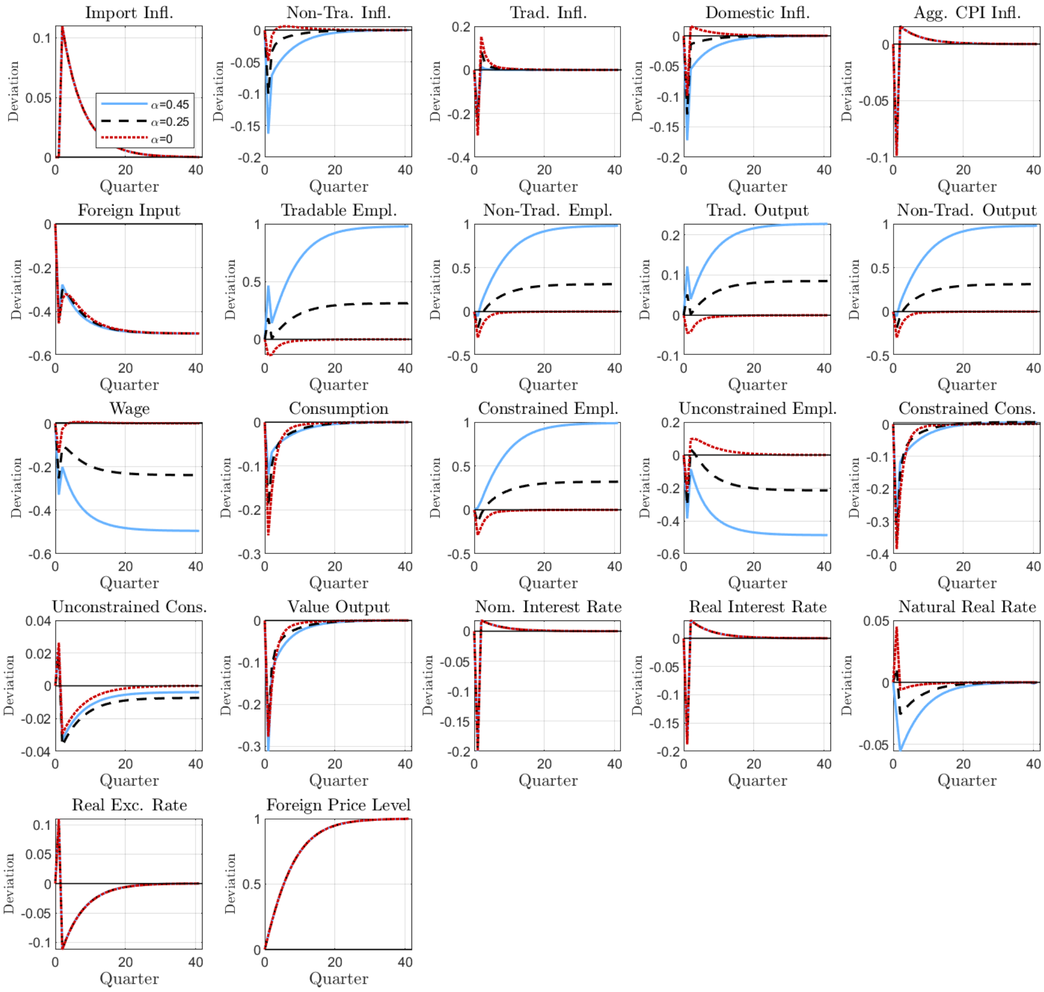
The degree of home bias in consumption appears to play a crucial role. Economies that are more open tend to be more exposed to fluctuations in foreign prices, which affects their consumption and production responses. In the scenarios involving sustained increases in foreign prices, whether gradual or front-loaded, the more open economy experiences a more pronounced decline in the natural rate of interest. Conversely, this trend reverses in the event of a negative TFP shock. Although the shock primarily affects the tradable sector, it directly impacts domestic production and consumption, significantly affecting the consumption basket of less open economies. In contrast, more open economies can mitigate the impact by diversifying away from domestic shocks through trade with foreign suppliers (and buyers).

**Gradual Fragmentation** Figure 10 shows that the open economies ($\alpha = 0.25$ and $\alpha = 0.45$) respond differently from the closed economy ($\alpha = 0$), in a gradual fragmentation scenario. Aggregate inflation has the same behaviour in all three cases; however, the underlying mechanisms differ quantitatively.

A useful benchmark to start with is the closed economy case ($\alpha = 0$, red dotted lines). Here, imported inputs are only used as an intermediate input for the production of non-tradable goods and the consumption basket of domestic households consists solely of domestically produced tradable and non-tradable goods. As the price of imported intermediate inputs increases, imported inflation increases and the use of the imported input falls. Non-tradable employment falls as non-tradable output falls. Employment falls for both types of households and real wages fall. The adverse effect on labour income leads to a fall in consumption for both types of households. The fall in aggregate consumption, which is almost entirely attributable to the fall in constrained households' consumption, leads to downward pressure on tradable output, tradable goods inflation, and domestic inflation.

Next, consider the case of an open economy ($\alpha = 0.25$ and $\alpha = 0.45$, dashed black lines and solid blue lines), where imported goods are not only used as intermediate inputs in the production of non-tradable goods but also comprise part of the consumption basket. More openness leads to a larger domestic adjustment in response to the import price shock. The additional fall in non-tradable goods inflation is due to the fall in domestic consumption, which now exerts additional downward pressure on domestic inflation. More openness also leads to a larger domestic adjustment in response to the import price shock. The import price shock leads households to substitute towards relatively cheaper domestically produced goods and production factors, which stimulates domestic demand and labour demand. Employment and output in both the tradable and non-tradable sectors increase. As exposure to foreign shocks intensifies with a lower home bias (higher $\alpha$), the natural rate of interest falls by more in the open economies.

Figure 10: IRFs to a Gradual Increase in Foreign Price Level.



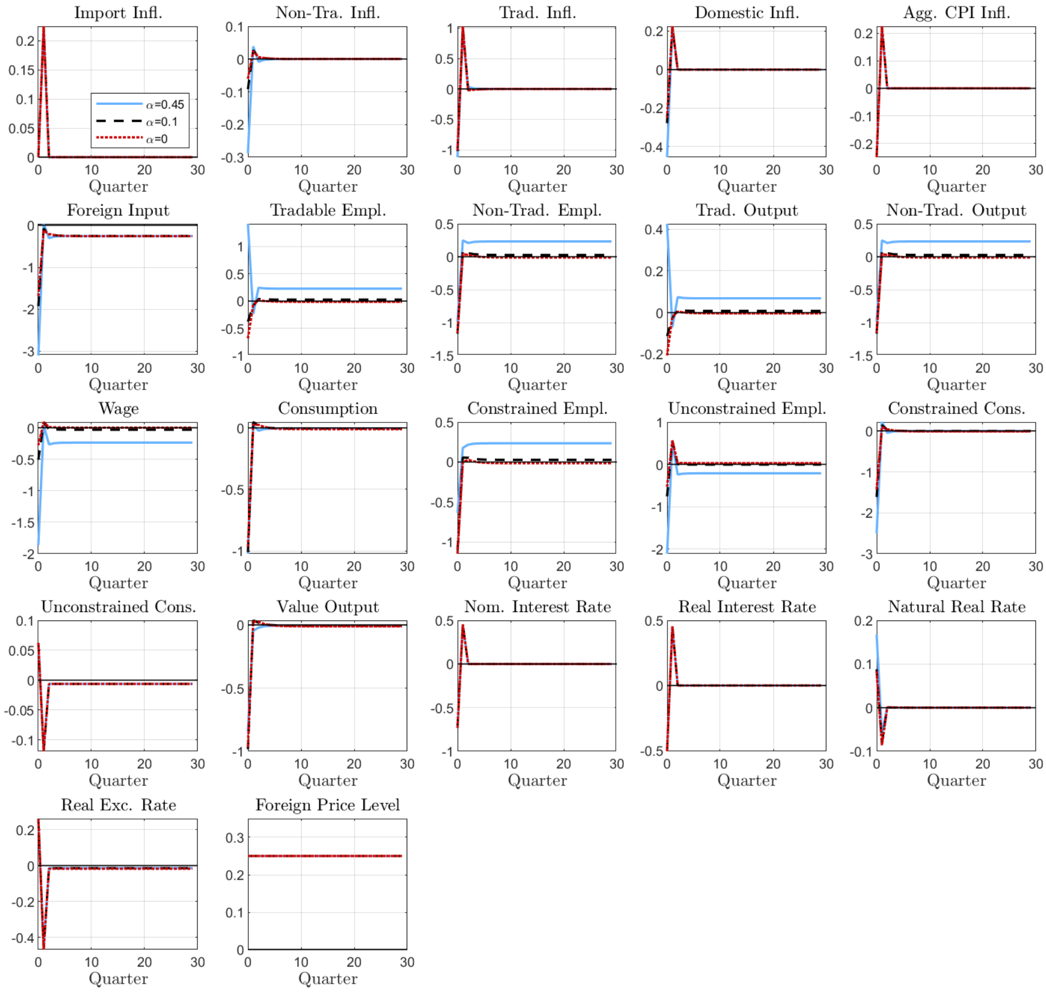

*Notes:* $\alpha = 0$ in red dotted line, $\alpha = 0.25$ in black dashed line, $\alpha = 0.45$ in blue line.

**Front-loaded Fragmentation** In the front-loaded fragmentation scenario, openness also plays a significant role in the responsiveness to changes in the level of $P_{F,t}$ (Figure 11). Notably, in the more open economy, non-tradable inflation exhibits a more pronounced decline immediately following the shock, contributing to a greater fall in domestic inflation.

Consider the case of a closed economy, where the import price shock only affects intermediate inputs in non-tradables production ($\alpha = 0$). As the price of foreign inputs rises, the economy experiences a decrease in demand for these inputs, leading to a fall in non-tradable output, a fall in non-tradable employment, a fall in real wages, and a fall in consumption. The fall in aggregate consumption is mostly attributable to the fall in constrained households' consumption. Tradable output and employment also fall, with tradable inflation falling as a result.

The same dynamics exist in the open economies ($\alpha = 0.25$ and $\alpha = 0.45$, dashed black lines and solid blue lines). However, imported goods are now also part of the consumption basket. The import price shock therefore stimulates the domestic economy, and to a greater degree in the more open economy, as households substitute towards relatively cheaper domestically produced goods. Output and employment increase in both the tradable and non-tradable sectors. The fall in consumption, which would occur as a result of more expensive foreign goods, is mitigated by the increase in domestic demand and labour income.

Figure 11: IRFs to a Front-loaded Increase in Foreign Price Level.



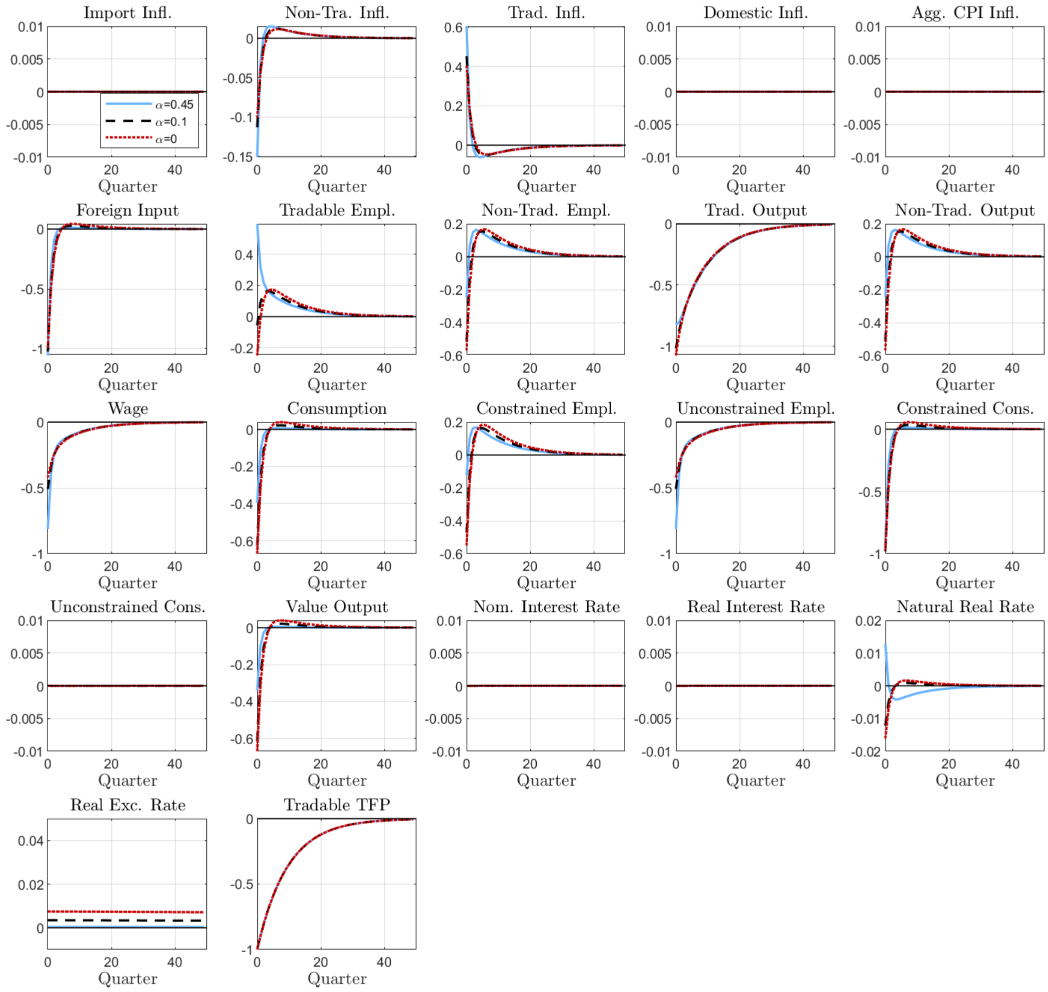

*Notes:* $\alpha = 0$ in red dotted line, $\alpha = 0.25$ in black dashed line, $\alpha = 0.45$ in blue line.

**Fall in Tradables Productivity** $A_{T,t}$ Figure 12 shows the impulse response functions for a one-standard deviation fall in TFP for the tradable sector. As before, consider first the $\alpha = 0$ specification (red dotted lines). In this case, imported inputs are only used in the production of non-tradable goods. An adverse productivity shock in the tradables sector leads to a fall in tradable output. Each unit of tradable output now requires more labour to produce, and hours worked increase gradually due to higher labour demand. The adverse TFP shock leads to an increase in marginal costs in the tradable sector, which leads to upward pressure on tradable inflation initially. Labour supply increases, leading to a fall in real wages through a fall in the nominal wage. The fall in real wages leads to a fall in consumption, which is entirely driven by a fall in constrained households' consumption, as unconstrained households can borrow to smooth over the transitory shock. Constrained households are unable to borrow to smooth consumption, increasing labour supply instead.

The fall in consumption affects the non-tradable sector, where output falls. However, since the non-tradable sector is unaffected by the adverse productivity shock, it can increase output by substituting towards labour as real wages fall. Non-tradable output rises as a result, through an increase in the intensive margin of labour and a fall in the use of the foreign input. The increase in tradable inflation counteracts the fall in non-tradable inflation and domestic inflation remains unchanged as a result. As this is purely a domestic shock, there is no change in imported inflation or in aggregate CPI inflation as a result.

In a more open economy (and $\alpha = 0.45$), the adverse effect the productivity shock on overall consumption is smaller on impact, and tradable output falls by less as the use of the foreign input falls by less than in the closed economy. Employment in the tradable sector increases significantly, compensating for part of the losses in productivity (given the Cobb-Douglas specification, scope for labour substitutability is limited). The fall in productivity leads to a large fall in the real wage on impact. As in the $\alpha = 0$ case, the adverse productivity shock in the tradables sector has some spillovers for the non-tradables sector, although this effect is not as large. Despite a significant fall in real wages, non-tradable employment and output increase by roughly the same magnitude as in the $\alpha = 0$ case. Changes in tradable inflation are balanced by countervailing changes in non-tradable inflation, with domestic and aggregate CPI inflation unchanged as a result.

Figure 12: IRFs to a Negative Shock to Tradable TFP.




*Notes:* $\alpha = 0$ in red dotted line, $\alpha = 0.25$ in black dashed line, $\alpha = 0.45$ in blue line.

# 4 Extensions

In this section, we extend the RANK model introduced in Section 3.1 to incorporate nominal wage rigidities.

## 4.1 Households

The setting is similar to the RANK model we discuss before. The key difference we introduce in this section is that, once, households have chosen how much labour to supply in a given period, $N_t(j)$, this labour is supplied to a union, in return for a nominal wage $W_t$. The union allocates the workers into categories or varieties indexed by $j$, $N_t(j), j \in [0,1]$ and sells these units of labour varieties at wage $W_t(j)$. Owing to imperfect substitutability, the union can act as a monopolist over each variety.

**Labour Packers** Varieties $N_t(j)$ are in turn combined by labour "packers" according to a CES production function:

$$N_t = \left[\int_0^1 N_t(j)^{1-\frac{1}{\epsilon_w}} dj\right]^{\frac{1}{1-\frac{1}{\epsilon_w}}}$$

$N_t(j)$ denotes the demand for a specific labour variety $j$ and $N_t$ denotes aggregate labour demand. $\epsilon_w$ is the elasticity of substitution between labour varieties. After the packers have assembled the labour bundle they supply it at wage $W_t$ to firms that then use it in the production process. The Labour packers maximise the following objective:

$$\max_{N_t(j)} \left\{W_t N_t - \int_0^1 N_t(j) W_t(j) dj\right\}$$

subject to the CES production function. The FOC is:

$$W_t N_t^{\frac{1}{\epsilon_w}} N_t(j)^{-\frac{1}{\epsilon_w}} - W_t(j) = 0$$

Therefore the labour demand is given by:

$$N_t(j)^d = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t$$

As the labour packers are perfectly competitive we can use the zero-profit condition to calculate the wage index:

$$W_t N_t - \int_0^1 N_t(j) W_t(j) dj = 0 \implies$$

$$W_t N_t = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t W_t(j) dj$$

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj\right)^{\frac{1}{1-\epsilon_w}}$$

**Labour Unions** The union maximises:

$$\max_{W_t(j)} \delta_t N_t^d W_t(j) - \frac{N_t^{1+\phi}}{1+\phi}$$

subject to:

$$N_t(j)^d = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t^d$$

This is equivalent to the household maximisation of utility subject to the budget constraint and the demand for labour.

The FOC becomes:

$$\delta_t \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} (1-\epsilon_w) N_t^d + \epsilon_w \left[\left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t^d\right]^{\phi} \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w - 1} \frac{N_t^d}{W_t} = 0$$

This yields:

$$W_t(j) = \frac{\epsilon_w}{\epsilon_w - 1} \frac{(N_t^d)^{\phi}}{C_t^{\sigma}}$$

where $\frac{\epsilon_w}{\epsilon_w-1} = \mathcal{M}^w$ and $mc_t^w = \frac{(N_t^d)^{\phi}}{C_t^{\sigma}}$. This represents the flexible wage setting. However, as we introduce *nominal and real wage stickiness*, we have that with probability $\theta_w$ a union is stuck with its previous-period wage indexed to a composite such that:

$$W_t(j) = \begin{cases} W_{t-1}(j) \left((\Pi_w^{ss})^{1-\xi_w} (\Pi_{t-1}^w)^{\xi_w}\right) & \text{with prob } \theta_w \\ W_t^*(j) & \text{with prob } 1-\theta_w \end{cases}$$

Therefore,

$$W_{t+s}(j) = W_t^*(j) \left(\Pi_{ss}^W\right)^{s(1-\xi_w)} \left(\prod_{g=0}^{s-1} \left(\left(\Pi_{t+g}^W\right)^{\xi_w}\right)\right) = W_t^*(j) \left[\left(\Pi_{ss}^W\right)^{s(1-\xi_w)} \left(\frac{W_{t+s-1}}{W_{t-1}}\right)^{\xi_w}\right]$$

Subject to the above-derived demand constraint and assuming that a union $j$ always meets the demand for its labour at the current wage labour unions solve the following optimisation problem

$$\max_{W_t^*(j)} E_t \sum_{s=0}^{\infty} (\theta_w)^s \Lambda_{t,t+s} P_{t+s} \left[\left(\frac{W_t^*(j)}{P_{t+s}} - mc_{t+s}^W\right) \left(\frac{W_t^*(j)}{W_{t+s}}\right)^{-\frac{M_w}{M_w - 1}} N_{t+s}\right] \tag{31}$$

Taking the derivative with respect to $W_t^*(j)$ delivers the familiar wage inflation schedule

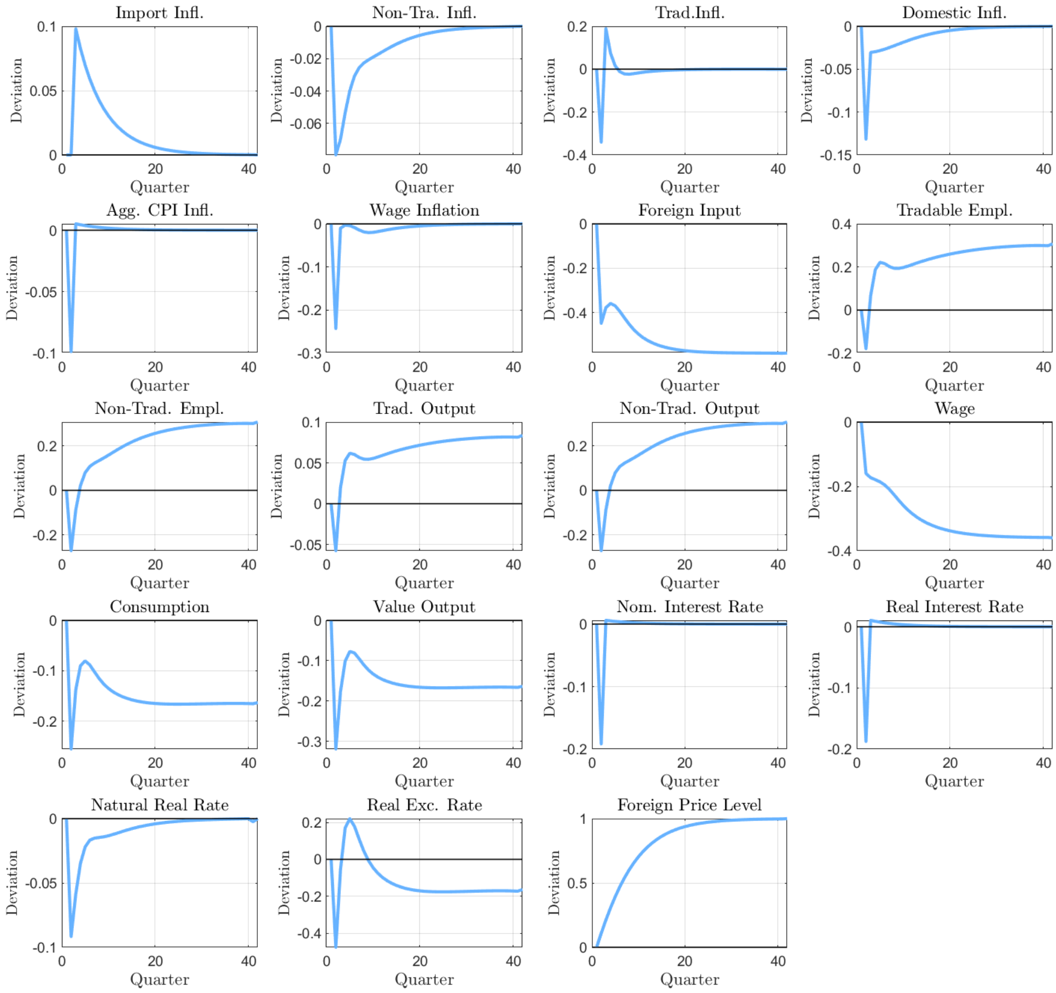
$$
\begin{aligned}
&\frac{f_t^{W,1}}{f_t^{W,2}}\mathcal{M}_w = w_t^* = \frac{W_t^*}{W_t} = \left(\frac{1-\theta_w\left(\zeta_t^W\right)^{\frac{1}{\mathcal{M}_w-1}}}{1-\theta_w}\right)^{1-\mathcal{M}_w} \\
&f_t^{W,1} = N_t\frac{mc_t^W}{w_t} + \theta_w E_t\left[(\Lambda_{t,t+1})\left(\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W}\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} f_{t+1}^{W,1}\right] \\
&f_t^{W,2} = N_t + \theta_w\beta E_t\left[(\Lambda_{t,t+1})\left(\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W}\right)^{\frac{1}{\mathcal{M}_w-1}} f_{t+1}^{W,2}\right] \\
&\zeta_t^W = \Pi_t^W/\Pi_{ss}^W \\
&w_t = \Pi_t^W/\Pi_t w_{t-1} \\
&\mathcal{D}_t^W = (1-\theta_w)\left(\frac{1-\theta_w\left(\zeta_t^W\right)^{\frac{1}{\mathcal{M}_w-1}}}{1-\theta_w}\right)^{\mathcal{M}_w} + \theta_w\left(\zeta_t^W\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\mathcal{D}_{t-1}^W
\end{aligned}
$$

Where $D_t^W$ is wage dispersion. We calibrate $\theta_w = 0.92$, $\xi_w = 0$, $\epsilon_w = 11$ as in Chan, Diz, and Kanngiesser (2024). All the other features of the model stay unchanged.

**Gradual Fragmentation** The impact of a gradual import price shock on aggregate CPI inflation and domestic inflation with wage rigidities (Figure 13) is similar to the case without wage rigidities (Figure 4). However, the composition of domestic inflation differs: there is a deeper fall in tradables inflation, and a shallower trough for non-tradables inflation. A salient feature of this extension is the initial fall in tradable output. Higher marginal costs lead both tradable and non-tradable goods firms to cut production. Hours worked fall in both the tradables sector and the non-tradables sector. Aggregate hours worked fall as a result and to a much greater degree than in the flexible wage case. Aggregate consumption falls to a greater extent initially, despite a more moderate fall in the real wage than the flexible wage case. Despite the fall in supply capacity for tradable goods, tradable inflation falls significantly, as consumption also falls to a greater degree than in the case without wage rigidities.

**Front-loaded Fragmentation** The differences between the flexible-wage and the sticky-wage cases are more salient in the front-loaded fragmentation scenario. Relative to the flexible-wage case (Figure 5), nominal wage rigidities mitigate the fall in wage inflation (Figure 14). As aggregate price inflation falls to a greater degree, real wages increase. The increases in both labour costs and intermediate input costs lead to a fall in tradable and non-tradable output. Hours worked fall in both the tradables sector and the non-tradables sector, leading to a fall in aggregate hours worked (and to a much greater degree than in the flexible-wage case). Aggregate consumption falls despite an increase in the real wage.

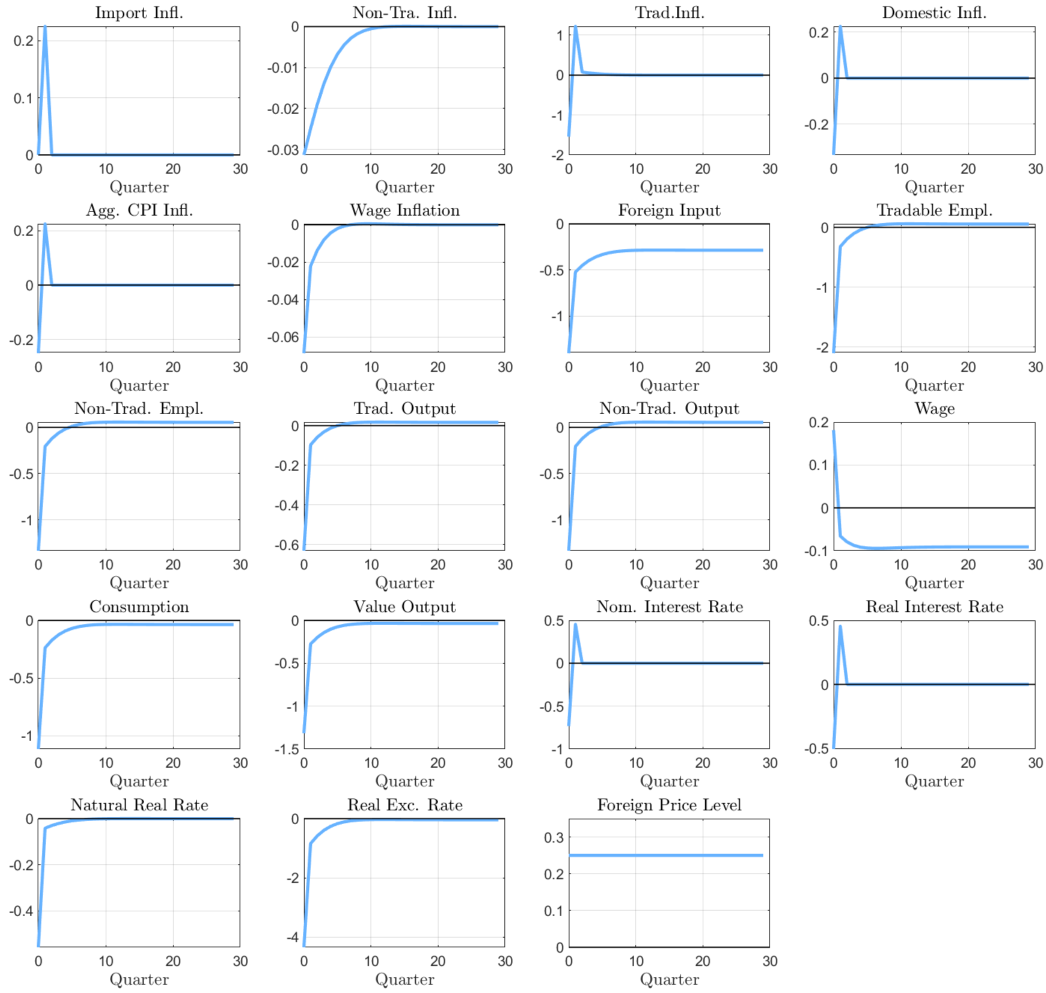
Figure 13: IRFs to a Gradual Increase in Foreign Price Level.




*Notes:* IRFs to a 100% gradual positive foreign price shock.

While the impact of a front-loaded import price shock on aggregate CPI inflation and domestic inflation is similar to the case without wage rigidities, the fall in domestic inflation is mostly due to the fall in tradables inflation instead, as non-tradable inflation falls by a negligible amount. Again, the fall in tradable output is a salient feature of this extension. Higher labour costs lead tradable goods firms to cut production. Despite the fall in supply capacity for tradable goods, tradable inflation falls significantly, as consumption also falls to a greater degree than in the case without wage rigidities. The real exchange rate appreciates, resulting in a significant fall in exports and a smaller fall in imports, relative to the flexible wage case.

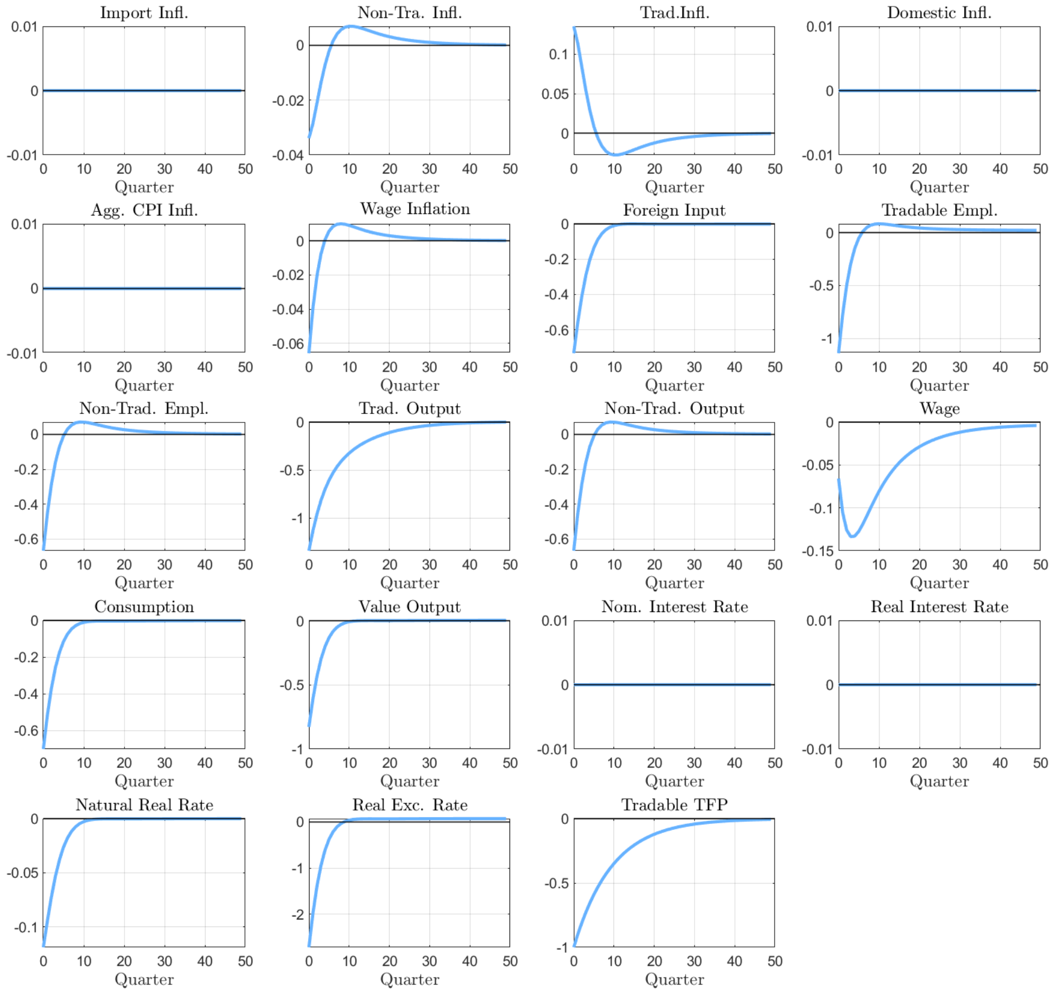
Figure 14: IRFs to a Front-loaded Increase in Foreign Price Level.




*Notes:* IRFs to a 25% positive foreign price shock.

**Fall in Tradables Productivity $A_{T,t}$.** Figure 15 shows the responses to an adverse productivity shock in the tradable goods sector. As in the case of an import price shock, nominal wage rigidities mitigate the fall in wages that would occur in the presence of flexible wages, resulting in a smaller decrease in the real wage. The increase in marginal costs leads to a sharper fall in tradable output. Relative to the flexible-wage case, tradable employment falls instead. In addition, non-tradable output now falls, as labour costs have increased for both sectors. Aggregate consumption falls by more in the sticky-wage case, despite a much shallower fall in the real wage. Tradable inflation still increases due to supply constraints in this sector, yet the substantial fall in aggregate demand mitigates its rise.

Figure 15: IRFs to a Negative Shock to Tradable TFP.




*Notes:* IRFs to a negative TFP shock in the tradable sector.

# 5 Conclusion

There is a broad consensus that a realignment of trading patterns is taking place. While trade fragmentation will likely lead to higher imported goods prices and lower real incomes, we show that the inflationary impact and the appropriate monetary policy response depend crucially on how demand adjusts to lower incomes, which in turn depends on the shape of the fragmentation process.

We study the macroeconomic effects of fragmentation using a two-sector, open economy New Keynesian model featuring imperfect risk sharing and heterogeneous households. We consider different fragmentation scenarios by varying the speed at which foreign prices adjust to a permanently higher level, as well as a negative shock to tradable sector productivity. The scenarios we consider capture the usual supply-side channels whereby higher input prices affect inflation, but emphasize how the overall effect on inflation depends on the adjustment of demand to lower real incomes. The balance between supply and demand differs in each of the scenarios, which has implications for inflation, and hence, monetary policy.

Our results suggest that trade fragmentation is not necessarily inflationary. In our first scenario, the gradual and permanent increase in foreign prices yields a persistent increase in imported inflation; the pass-through to aggregate inflation is counteracted by a fall in domestic inflation. The key reason for this result is that forward-looking households reduce their spending in anticipation of the more restricted future supply, as they respond to lower permanent income by smoothing consumption. The natural rate of interest decreases, suggesting that, when we allow for demand to adjust, the overall effect is not inflationary. The economy enters a period of stagnation, with lower real incomes, lower demand and lower inflationary pressures. In our second scenario, a front-loaded and persistent increase in the price of imported goods leads to a temporary tradeoff or stagflation, with lower consumption and higher inflationary pressures. Finally, a fall in the productivity of tradable goods leads to lower tradable output and an increase in tradable inflation, countervailed by a fall in non-tradable inflation driven by a fall in consumption and an increase in labour supply.

To understand the various channels shaping the response of demand and inflation to fragmentation, we vary the proportion of constrained households and the home bias in these three scenarios. We find that as the share of hand-to-mouth agents in the economy increases, demand falls by less in the gradual fragmentation scenario, as fewer unconstrained households anticipate the future fall in real incomes. More open economies are more exposed to shocks in foreign prices, which is reflected in the response of consumption and production. In both the front-loaded and gradual increase in imported prices scenarios, we see an amplification of the effects. However, this reverts in the case of a negative TFP shock, which is a domestic shock. In this scenario, more openness allows the economy to diversify away the impact of a domestic shocks on the economy, dampening the impact of the shock. Wage rigidities introduce additional supply-side constraints through the domestic labour market, leading to a fall in output in both sectors, with implications for the composition of domestic inflation.

In future work, we plan to formalise the policy tradeoffs posed by fragmentation, study optimal monetary policy, and explore settings with higher inertia in price setting and lags in the transmission of monetary policy.

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# A Appendix: Model details

Appendix A.1 contains the derivation of some of the main equations under specific assumptions of the parameters. Appendix A.2 contains the relevant model equations for our exercise.

## A.1 Derivations

Assuming $\sigma = \eta = \nu = 1$, we get:

$$C_t \equiv \frac{C_{H,t}^{1-\alpha} C_{F,t}^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{32}$$

$$C_{H,t} \equiv \frac{C_{N,t}^{1-\gamma} C_{t,t}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \tag{33}$$

$$P_t \equiv P_{H,t}^{1-\alpha} P_{F,t}^{\alpha} \tag{34}$$

$$P_{H,t} \equiv P_{N,t}^{1-\gamma} P_{T,t}^{\gamma} \tag{35}$$

We can write the aggregate domestic prices as follows:

$$P_t = P_{H,t}^{1-\alpha} P_{F,t}^{\alpha} = (P_{N,t}^{1-\gamma} P_{T,t}^{\gamma})^{1-\alpha} P_{F,t}^{\alpha} \tag{36}$$

Finally, we can derive the necessary demand functions:

$$C_{H,t} = (1-\alpha)\left(\frac{P_t}{P_{H,t}}\right) C_t = (1-\alpha)\mathcal{T}_t^{\alpha} C_t$$

$$C_{F,t} = \alpha\left(\frac{P_t}{P_{F,t}}\right) C_t = \alpha \mathcal{T}_t^{\alpha-1} C_t$$

$$C_{n,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_{n,t}}\right) C_{H,t}$$

$$C_{T,t} = \gamma\left(\frac{P_{H,t}}{P_{T,t}}\right) C_{H,t}$$

$$C_{N,t}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t}}\right)^{-\epsilon} C_{N,t}$$

For clarity, I am going to derive all the price dependencies below. From equation (34), we get:

$$P_{H,t} = \left(\frac{P_t}{P_{F,t}^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

$$P_{F,t} = \left(\frac{P_t}{P_{H,t}^{1-\alpha}}\right)^{\frac{1}{\alpha}}$$

$$\mathcal{T}_t^{-\alpha} = \frac{P_{H,t}}{P_t}$$

$$\mathcal{T}_t^{1-\alpha} = \frac{P_{F,t}}{P_t}$$

Now using equation (35):

$$
\begin{aligned}
&\mathcal{T}_t^{-\alpha} = \frac{P_{H,t}}{P_t} = \frac{P_{N,t}^{1-\gamma} P_{T,t}^{\gamma}}{P_t} \\
&\mathcal{T}_t^{-\alpha} = \frac{P_{N,t} P_{N,t}^{-\gamma} P_{T,t}^{\gamma}}{P_t} = \frac{P_{N,t}}{P_t}\left(\frac{P_{T,t}}{P_{N,t}}\right)^{\gamma} \implies \frac{P_{N,t}}{P_t} = \mathcal{T}_t^{-\alpha}\left(\frac{P_{N,t}}{P_{T,t}}\right)^{\gamma} \\
&\mathcal{T}_t^{-\alpha} = \frac{P_{N,t}^{1-\gamma} P_{T,t}^{\gamma}}{P_t} = \frac{P_{N,t}^{1-\gamma} P_{T,t}^{\gamma}}{P_t}\frac{P_{T,t}}{P_{T,t}} = \frac{P_{N,t}^{1-\gamma} P_{T,t}^{\gamma-1}}{P_t} P_{T,t} = \left(\frac{P_{N,t}}{P_{T,t}}\right)^{1-\gamma}\frac{P_{T,t}}{P_t} \implies \frac{P_{T,t}}{P_t} = \mathcal{T}_t^{-\alpha}\left(\frac{P_{T,t}}{P_{N,t}}\right)^{1-\gamma}
\end{aligned}
$$

Tradable are by definition consumed both at home and outside:

$$
\begin{aligned}
C_{T,t} &= \gamma\left(\frac{P_{H,t}}{P_{t,t}}\right) C_{H,t} = \gamma\left(\frac{P_{H,t}}{P_{t,t}}\right)(1-\alpha)\left(\frac{P_t}{P_{H,t}}\right) C_t = \gamma(1-\alpha)\left(\frac{P_t}{P_{T,t}}\right) C_t \\
C_{T,t}^* &= \gamma\alpha\left(\frac{P_t^*}{P_{t,t}^*}\right) C_t^* = \gamma\alpha\left(\frac{P_t^*\mathcal{E}_t}{P_{T,t}}\frac{P_t}{P_t}\right) C_t^* = \gamma\alpha\left(\frac{P_t}{P_{T,t}}\right)\mathcal{S}_t C_t^* \\
Y_{T,t} &= C_{T,t} + C_{T,t}^* = \gamma\left(\frac{P_t}{P_{T,t}}\right)[(1-\alpha)C_t + \alpha\mathcal{S}_t C_t^*]
\end{aligned}
$$

For the non-tradable:

$$
\begin{aligned}
C_{N,t} &= (1-\gamma)\left(\frac{P_{H,t}}{P_{n,t}}\right) C_{H,t} \\
C_{N,t} &= (1-\gamma)\left(\frac{P_{H,t}}{P_{n,t}}\right)(1-\alpha)\left(\frac{P_t}{P_{H,t}}\right) C_t \\
C_{N,t} &= (1-\alpha)(1-\gamma)\left(\frac{P_t}{P_{n,t}}\right) C_t
\end{aligned}
$$

Therefore,

$$
Y_t = (1-\alpha)(1-\gamma)\left(\frac{P_t}{P_{n,t}}\right) C_t + \gamma\left(\frac{P_t}{P_{T,t}}\right)[(1-\alpha)C_t + \alpha\mathcal{S}_t C_t^*]
$$

Further, to derive the natural level of output, we use our assumption on $\sigma = \eta = 1$, and we get:

$$
\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)\right] = \left(\frac{C_t^*}{C_{t+1}^*}\right)\frac{\mathcal{S}_t}{\mathcal{S}_{t+1}}\left[1 + \chi(b_t^* - \bar{b}^*)\right]
$$

$$
\begin{aligned}
Y_t^n = \Sigma_{\alpha\gamma,t}\mu^{\frac{1}{1-\kappa}} A_{N,t}^{\frac{1}{1-\kappa}} p_{N,t}^{\frac{1}{1-\kappa}} \Gamma^{-\frac{1}{1-\kappa}} p_{F,t}^{-\frac{\kappa}{1-\kappa}} \Bigg[ & \left( A_{T,t}^{-\frac{1}{1-\zeta}}\left(\gamma\left(\frac{P_t}{P_{T,t}}\right)((1-\alpha)\mathcal{D}_t + \alpha)\,\mathcal{S}_t C_t^*\right)^{\frac{1}{1-\zeta}}\right. \\
& \left. + (1-\alpha)(1-\gamma)\left(\frac{P_t}{P_{N,t}}\right)\mathcal{D}_t\mathcal{S}_t C_t^*\left(\frac{p_{F,t}}{p_{N,t}}\right)^{\frac{1-\kappa}{\kappa}}\kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}}\right)^{-\phi}\Bigg]
\end{aligned}
$$

## A.2 Appendix: Summary of Model Equations – TANK

Note: Throughout this appendix and in the Dynare code, I express everything in relative terms to Aggregate Domestic CPI $P_t$. This means the following:

$$p_{j,t} = \frac{P_{j,t}}{P_t} \quad \forall j = \{T, F, N, H\}$$
$$w_t = \frac{W_t}{P_t}$$

I can rewrite the exchange rate in terms of foreign prices as follows:

$$p_{F,t} = \frac{P_{F,t}}{P_t}$$

**Households**: For the unconstrained households we have the marginal utility of consumption (A.1), the budget constraint (A.2), the intertemporal substitution of consumption conditions (A.3), the foreign bonds condition (A.4), and the intratemporal substitution between labour and consumption (A.5). For the constrained household, we only have the budget constraint (A.6), because they do not have access to the bond market, so they can't smooth consumption. Finally, we have the heterogeneity index (A.7) and total consumption (A.8).

$$\delta_t = (C_t^U)^{-\sigma} \tag{A.1}$$

$$C_t^U + b_t + \mathcal{S}_t b_t^* = b_{t-1}\frac{(1+i_{t-1})}{(1+\pi_t)} + \mathcal{S}_t b_{t-1}^* \frac{(1+i_{t-1}^*)}{(1+\pi_t^*)} + \frac{W_t}{P_t} N_t^U - \frac{\chi}{2}\mathcal{S}_t \left(b_t^* - \bar{b}^*\right)^2 \tag{A.2}$$

$$\frac{1}{(1+i_t)} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1}{(1+\pi_{t+1})} \right] \tag{A.3}$$

$$[1 + \chi(b_t^* - \bar{b}^*)] = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}^U}{C_t^U} \right)^{-\sigma} \frac{1+i_t^*}{(1+\pi_{t+1}^*)} \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} \right] \tag{A.4}$$

$$(N_t^U)^{\phi}(C_t^U)^{\sigma} = w_t \tag{A.5}$$

$$C_t^C = w_t N_t^C \tag{A.6}$$

$$\Gamma_t = \frac{C_{u,t}}{C_t} \tag{A.7}$$

$$C_t^U = \frac{1}{(1-\lambda)}(C_t - \lambda C_t^C) \tag{A.8}$$

**Firms**:

**Non Tradable**: We have a technology equation (A.9), production factor demand, respectively demand for foreign input (A.10) and for non-tradable labour (A.11), the non-tradable

Phillips Curve (A.12) and the Non-Tradable Domestic Demand (A.13).

$$Y_{N,t} = A_{N,t} M_{F,t}^{\kappa} N_{N,t}^{1-\kappa} \tag{A.9}$$

$$p_{F,t} M_{F,t} = mc_t \kappa Y_{N,t} \tag{A.10}$$

$$w_t N_{N,t} = mc_t (1-\kappa) Y_{N,t} \tag{A.11}$$

$$\Pi_{N,t} \left( \Pi_{n,t} - \bar{\pi} \right) = \beta \mathbb{E}_t \left[ \frac{\delta_{t+1}}{\delta_t} \Pi_{n,t+1} \left( \Pi_{N,t+1} - \bar{\pi} \right) \frac{Y_{N,t+1}}{Y_{n,t}} \right] + \frac{\epsilon}{\xi} \left( mc_t - \frac{\epsilon - 1}{\epsilon} \right) \tag{A.12}$$

$$Y_{N,t} = (1-\alpha)(1-\gamma) \left( \frac{C_t}{p_{N,t}} \right) \tag{A.13}$$

**Tradable**: We have a technology equation (A.14), employment demand (A.15), and total tradable demand (A.16). We do not have a price-setting equation because they take the price as given by the international market.

$$Y_{T,t} = A_{T,t} N_{T,t}^{1-\zeta} \tag{A.14}$$

$$w_t N_{T,t} = (1-\zeta) Y_{T,t} p_{T,t} \tag{A.15}$$

$$Y_{T,t} = \left( \frac{\gamma}{p_{T,t}} \right) \left[ (1-\alpha) C_t + \alpha \mathcal{S}_t C_t^* \right] \tag{A.16}$$

Total Profits are given by:

$$\Psi_t = p_{T,t} Y_{T,t} - w_t N_{T,t} + p_{N,t} Y_{N,t} \left( 1 - \frac{\xi}{2} \left( \Pi_n - \bar{\pi} \right)^2 \right) - p_{F,t} M_{F,t} - w_t N_{N,t} \tag{A.17}$$

**Market Clearing:** Goods market clearing (A.18), Labour market clearing (A.19), Bond Market clearing (A.20), Foreign demand (A.21).

$$Y_t = p_{T,t} Y_{T,t} + p_{N,t} Y_{N,t} \left( 1 - \frac{\xi}{2} \left( \Pi_n - \bar{\pi} \right)^2 \right) \tag{A.18}$$

$$N_t^C \lambda + N_t^U (1-\lambda) = N_{N,t} + N_{T,t} \tag{A.19}$$

$$b_t^H = 0 \tag{A.20}$$

$$Y_t^* = C_t^* \tag{A.21}$$

**Prices:** CPI is given by (A.22), inflation for non-tradables (A.23) and tradables (A.24), imported inflation (A.25) and finally real depreciation (A.28).

$$1 = (p_{N,t}^{1-\gamma} p_{T,t}^{\gamma})^{1-\alpha} p_{F,t}^{\alpha} \tag{A.22}$$

$$\pi_{N,t} = \frac{p_{N,t}}{p_{N,t-1}} \pi_t \tag{A.23}$$

$$\pi_{T,t} = \frac{p_{T,t}}{p_{T,t-1}} \pi_t \tag{A.24}$$

$$\pi_{F,t} = \frac{p_{F,t}}{p_{F,t-1}} \pi_t \tag{A.25}$$

$$p_{H,t} = p_{N,t}^{(1-\gamma)} p_{T,t}^{\gamma} \tag{A.26}$$

$$\pi_{H,t} = \frac{p_{H,t}}{p_{H,t-1}} \pi_t \tag{A.27}$$

$$\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \Delta \mathcal{E}_t \frac{\Pi_t^*}{\Pi_t} \tag{A.28}$$

**Policy Rule:** Note that $1 + i_t = R_t$.

$$\frac{R_t}{R_{ss}} = \left(\frac{\Pi}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y}{\bar{Y}}\right)^{\phi_y} \exp(\nu_m) \tag{A.29}$$

**Natural real rate:**

$$(1 + r_t^n) = \frac{1}{\beta} \left( \frac{Y_{t+1}^n}{Y_t^n} \frac{\Sigma_{\alpha\gamma,t}}{\Sigma_{\alpha\gamma,t+1}} \right) \tag{A.30}$$

**AR (1) processes:** In order, for non-tradable productivity, tradable productivity, foreign demand, foreign tradable prices and inflation and foreign interest rate.

$$A_{N,t} = (A_{n,ss})^{1-\rho_{an}} (A_{N,t-1})^{\rho_{an}} \exp(\nu_{an,t}) \tag{A.31}$$

$$A_{T,t} = (A_{T,ss})^{1-\rho_{at}} (A_{T,t-1})^{\rho_{at}} \exp(\nu_{at,t}) \tag{A.32}$$

$$C_t^* = (C_{t,ss}^*)^{1-\rho_{y^*}} (C_{t-1}^*)^{\rho_{y^*}} \exp(\nu_{y^*,t}) \tag{A.33}$$

$$\log \Pi_{F,t} = \rho_F \left(\log p_{F,t-1} - \log p_{F,t-2} - \log p_{F,ss}\right) + \nu_{\Pi_f,t} \tag{A.34}$$

$$i^* = (i_{ss}^*)^{1-\rho_i} (i_{t-1}^*)^{\rho_i} \exp(\nu_{i,t}) \tag{A.35}$$

$$\Pi_t^* = 1 \tag{A.36}$$

**Foreign Price Processes MA(53)**: The process for $P_{F,t}$ for immediate shocks:

$$\log(P_{F,t}) = \sum_{j=1}^{53} \theta_j \nu_{P_{F,t-j}} \tag{A.37}$$

**Gradual Steady State Change:** The process for $P_{F,t}$ is:

$$\log(P_{F,t}) = \log(P_{F,t-1}) + \rho_{P_F}(\log(P_{F,ss}(-\log(P_{F,t-1})) + \nu_{P_{F,t}} \tag{A.38}$$

**Auxiliary Variables**: Real Interest rate (17), nominal imports (A.40) and exports (A.41), natural level of output (A.42).

$$RR_t = \frac{(1+i_t)}{\Pi_{t+1}} \tag{A.39}$$

$$IM_t = p_{F,t}(M_{f,t} + C_{f,t}) \tag{A.40}$$

$$X = p_{T,t}\gamma\alpha\mathcal{S}_t C_t^* \tag{A.41}$$

$$\begin{aligned} Y_t^n = \Sigma_{\alpha\gamma,t} & \left(\mu^{\frac{1}{1-\kappa}} A_{N,t}^{\frac{1}{1-\kappa}} p_{N,t}^{\frac{1}{1-\kappa}} \Gamma^{-\frac{1}{1-\kappa}} p_{F,t}^{-\frac{\kappa}{1-\kappa}}\right)^{\frac{1}{\sigma}} \\ & \left(A_{T,t}^{-\frac{1}{1-\zeta}}\left(\left(\gamma(1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\mathcal{D}_t^{\frac{1}{\sigma}} + \gamma\alpha\left(\frac{P_t}{P_{T,t}}\right)^{\eta}\right)\right)\mathcal{S}_t C_t^*\right)^{\frac{1}{1-\zeta}} \\ & + (1-\alpha)(1-\gamma)\left(\frac{P_{H,t}}{P_{N,t}}\right)^{\nu}\left(\frac{P_t}{P_{H,t}}\right)^{\eta}\mathcal{D}_t^{\frac{1}{\sigma}}\mathcal{S}_t C_t^*\left(\frac{p_{F,t}}{p_{N,t}}\right)^{\frac{1-\kappa}{\kappa}}\kappa^{-\frac{\kappa}{1-\kappa}} A_{N,t}^{-\frac{1}{1-\kappa}}\left.\right)^{-\phi}\Bigg]^{\frac{1}{\sigma}} \end{aligned} \tag{A.42}$$