Granular Expectation Shocks and International Financial Contagion

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Abstract

Using a unique dataset linking investors' cross-country GDP growth expectations to their investments into mutual funds and to the mutual funds' cross-country allocation, we show that, while the flows into the funds are sensitive to the investors' fund-specific aggregate expectations (computed using the fund's portfolio shares), the funds' allocation reacts less to the country-level expectations. This gives rise to "co-ownership spillovers", whereby negative expectations about a country in which a fund invests can adversely affect capital flows to the other countries that are part of the fund's portfolio. Using a portfolio choice model with delegated investment, we show that these results arise naturally from a sticky portfolio friction. However, these spillovers matter in the aggregate only if the portfolio shares are granular. Finally, using our dataset and our model, we quantify the aggregate implications of these spillovers and find that co-ownership spillovers account for one fourth to one third of the comovement in expectation-driven capital flows.

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1 Introduction

Why do asset prices and business cycles comove in emerging economies? This comovement has been attributed to correlated fundamentals, global financial cycles, and real and financial contagion.[1] This study specifically examines the latter, and focuses on the role of equity investments by mutual funds, which manage a significant portion of capital flows to emerging economies. Understanding how these intermediaries allocate their capital across countries, and whether this allocation is efficient, is critical.

More specifically, this paper studies whether changes in investments into mutual funds driven by investors' expectations generates comovements in capital flows across countries through "co-ownership spillovers".[2] Co-ownership spillovers can arise through the following mechanism. An investor can choose how much to invest in a variety of global, emerging, or regional mutual funds, which invest equity in different sets of countries, and they can also choose to invest in safer assets (cash or bonds). The investor controls how much capital is sent to the mutual funds, but not how the capital is allocated between the countries that are part of a fund's portfolio. Now suppose that the investor expects that one country's asset market is going to perform poorly. She will then take away capital from the funds that invest in that country. If the funds share the same expectations as the investor, and continuously update their portfolio shares, then the funds' capital will be reallocated to the other countries in the portfolio, and these countries will not be negatively affected. But if portfolio shares are sticky, the other countries will also undergo some capital retrenchment.

Using a unique dataset linking investors' cross-country GDP growth expectations to their investments into mutual funds and to the mutual funds' cross-country allocation, we show that while the flows into the funds are highly sensitive to the investors' fund-specific aggregate expectations (computed using the fund's portfolio shares), the funds' allocation reacts less and with a lag to the investor's country-level expectations. Using a simple delegated investment model, we show that this creates co-ownership spillovers, where negative expectations about one country that is part of a fund's portfolio can negatively impact investment in the countries that are part of the same portfolio, through the aggregate expectations.

But are these spillovers relevant at the aggregate level? That is, do they lead to a significant level of contagion? Our model shows that they do not necessarily do so. For instance, if the country-specific shocks to expectations (shocks that are uncorrelated across countries) average out in the aggregate, then these spillovers will be driven only by global shocks (shocks that are correlated across countries). In that case, the co-ownership spillovers

[1] See Forbes and Rigobon (2001), Karolyi (2003), Forbes (2012) and Rigobón (2019) for useful surveys.

[2] We borrow this expression from Jotikasthira et al. (2012).

are not inefficient, as they are driven by global shocks that are relevant for all countries. However, if some countries compose a disproportionate share of portfolios, then expectations shocks specific to these countries spill-over to the other countries because they affect the aggregate fund-level expectations in a non-negligible way. The granularity of fund shares will thus matter, as shown by Gabaix (2011).

Along these lines, we show formally that co-ownership spillovers relate to the granular residual of the investors' fund-specific aggregate expectations and to a key elasticity parameter that we estimate using our data. We then quantify the contribution of the co-ownership spillovers to the aggregate capital flows, using the estimated key elasticity and the effective country shares and expectations from the data. The co-ownership spillovers account for one fourth to one third of the variance of the comovement in expectation-driven capital flows in our sample. Interestingly, both large advanced countries and small emerging countries are typical recipients of these spillovers. Both large advanced and emerging countries, like the G7 and BRICS, are typical contributors. This channel of international financial contagion is different from the typical "funding" channels that have been documented so far, as it does not necessarily give rise to North-South transmission, but rather to a Large-Small one. As a result, some large emerging economies are important contributors and do not suffer from major spillovers, like China and South Korea.

We contribute to several strands of literature. First, we contribute to the large literature that examines how shocks, local or global, are transmitted by mutual funds. Coval and Stafford (2007) show that U.S. mutual funds redeem investments as a consequence of funding shocks that originate from their investor base, and that these forced redemptions significantly impact U.S. domestic equity prices. Jotikasthira et al. (2012) show that global funds, domiciled in developed markets, display the same forced trading behavior as US domestic funds. They show that this flow-induced trading has a significant effect on prices, country betas and return co-movement among emerging markets. In general, it has been established, using micro-evidence from mutual funds, that shocks to the investor base are an important driver of the comovement in emerging markets (Broner et al., 2006; Gelos, 2011; Raddatz and Schmukler, 2012; Puy, 2016). There is however scarce evidence on co-ownership spillovers and on their ability to generate contagion and undesired fluctuations in capital flows and asset prices. An exception is Jotikasthira et al. (2012), who identify co-ownership spillovers by calibrating their model to the data. We instead provide direct evidence for this phenomenon by using the investor-level expectations to identify these spillovers.

Second, we contribute to a growing literature estimating the elasticity of investments to real-life expectations using survey data. Vissing-Jorgensen (2003), Glaser and Weber (2005), Kézdi and Willis (2011) and Weber et al. (2012) focus on households' expectations and their stock holding behavior. Piazzesi and Schneider (2009) examine the role of expectations on the housing market and Malmendier and Nagel (2015) and Agarwal et al. (2022) investigate how inflation expectations affect households' portfolio choices. Giglio et al. (2021) use a survey administered to a large panel of wealthy retail investors to study the relation between the investors' beliefs and their trading activity, while Dahlquist and Ibert (2021) focus on large institutional investors. Finally, De Marco et al. (2021) study European banks' investments in sovereign bonds across the Euro area. To the best of our knowledge, we are the first to estimate how investors' beliefs affect the cross-country allocation of equity investments.

Finally, we contribute the literature that examines frictions in portfolio adjustment. Importantly, our model provides an simple mapping from the portfolio stickiness to the the relative elasticity of capital flows to the country-specific expectation and to the aggregate expectation. Hence, we find that mutual funds must update their portfolios every 5 months on average. Previous evidence of delayed portfolio adjustment has been based on imputed expectations (that is, expectations constructed from observables, such as past returns) or on the persistence of portfolios.[3] Our estimate is lower than the one to two-year spans that have been identified using macroeconomic data (see for instance Bacchetta and van Wincoop (2017)). Besides, we show that the lack of response of portfolio shares to the country-level expectations is higher for the category of funds that we identify as "active" and for global emerging funds.[4]

Section 2 discusses the data and Section 3 estimates the elasticity of investment into and out of the mutual funds to investor's expectations. Section 4 lays down a portfolio choice model with delegated investment and shows when co-ownership spillovers appear and matter on the aggregate. Section 5 identifies the elasticity that is relevant to co-ownership spillovers by establishing a mapping from model to data. Finally, Section 6 quantifies the the co-ownership spillovers.

2 Data

Our dataset matches an expectation dataset to an investor and mutual fund dataset.

[3]Bohn and Tesar (1996), Froot et al. (2001), find that international portfolio flows are highly persistent and strongly related to lagged returns, and more recently Bacchetta et al. (2020) test a delayed adjustment model using mutual fund data.

[4]Sticky portfolios can also result from the prevalence of strict portfolio mandates. He and Xiong (2013) explore the moral hazard origins of this prevalence.

2.1 Expectation dataset: Consensus Economics Data

For information about forecasts, we use data from Consensus Economics. Consensus Economics is a survey firm polling individual economic forecasters on a monthly frequency. The survey covers 51 advanced and emerging countries for a maximum time span between 1989 and 2021. Each month, forecasters provide estimates of several macroeconomic indicators for the current and the following year. From this data, we use the real GDP growth forecasts. The Consensus Economics data also provides the name of the institution for each forecast. We extract and clean this information, which enables us to match the real GDP growth forecasts to our investor and mutual fund dataset.

2.2 Investor and mutual fund dataset: EPFR Data

The EPFR dataset is widely used to study cross-country investments in equity and bond markets. EPFR captures 5-20% of market capitalization in equity and bonds for most countries. It is a representative sample, as shown in Jotikasthira et al. (2012), show a close similarity between the EPFR data and matched CRSP data in terms of assets under management and average returns. Miao and Pant (2012) compare portfolio flows generated using EPFR data to portfolio flows computed with BOP data. Only a subset of institution investors flows are captured by the EPFR data, so there are clearly level differences, but the EPFR funds flows correlated well with BOP capital flows into Emerging Markets.

The EPFR data consists of two different datasets. The first dataset decomposes the weekly changes in assets under management of the mutual fund into weekly flows into the fund, and the weekly change in assets under management due to valuation changes. We aggregate this information to match the monthly frequency of the forecast data from Consensus Economics and the funds monthly country allocations described below.

The second dataset is a monthly dataset that contains information about country allocations at the fund level, that is, the share of the total assets invested in each country, the share of total assets held in cash, and the total assets managed by the fund.

Both the weekly flow data and the monthly country allocation data contain information about the financial institution managing the fund. These fund managers are typically global banks, so we call them "investors". We use this information to match the investors' name to the institution reported by Consensus Economics. After dropping observations with no information on country allocations, we find a match for 31 different managers. The country allocations of the funds managed by those investors overlap with the forecast information of Consensus Economics for 44 countries. Note that many expectation data are missing, since we have expectations only for 23% of countries on average for the funds that belong to our matched dataset, and only for 30% of countries when countries are weighted by portfolios. This leads to some econometric issues that we will address below.

As of January 2016, there are 17,260 mutual funds managing 14.1 trillion USD in assets reporting the weekly flows to EPFR and 1,151 mutual funds managing a total of 1 trillion USD in assets reporting monthly allocations. Of the 1,151 mutual funds in the EPFR data, 237 funds managing 156 billion USD in assets are present in the matched EFPR sample with Consensus Economics. The funds that we manage to match to Consensus Economics dataset seem to represent well the rest of the sample. The distributions of assets under management and allocations in the whole EPFR Data and in our merged sample are similar.[5]

In our econometric analysis, we drop all country-fund pairs with less than 24 observations (the equivalent of two years of data), and all funds with less than 200 observations. In this dataset, we have 8 investors, 73 funds, 43 countries and 52000 observations.

3 Elasticity of Capital Flows to Expectations

The aggregate country allocation response to investor expectations will depend on how flows into the mutual funds respond to these expectations, and on how the mutual funds adjust their allocations across countries. We examine each in turn, and establish two main results. First, an increase in an investor' GDP growth expectations associated to a mutual fund portfolio is followed by a significant increase in the flows into that mutual fund. However, the mutual fund's country allocation responds mildly or insignificantly to that investor's country-specific GDP growth expectations, save for a small sub-sample of global emerging market funds.

3.1 Investor expectations and flows to mutual funds

Define the aggregate growth expectation at the investor and fund level as the average growth expectation weighted by the past country allocations:

$$E_t^i g_p^{j,\text{next year}} = \sum_{k \in K(i,j)} w_{k,t-1}^{i,j} E_t^i g_{k,t}^{\text{next year}}, \tag{1}$$

where $w_{k,t-1}^{i,j}$ is mutual fund j's allocation to country k in month $t-1$, and $E_t^i g_{k,t}^{\text{next year}}$ is investor i's month t GDP growth expectation for country k between the current year and the next, in percent. Subscript p denotes a portfolio-level growth expectation. $K(i,j)$ is the set of countries in which fund j and for which we observe expectations. The sum of the weights

[5] These results are available upon request.

$w_{k,t-1}^{i,j}$ do not necessarily sum to 1, because we do not observe expectations for all countries. This generates some identification issues that we will address below.

We run the panel fixed-effects regression,

$$\ln\left(A_t^{i,j}\right) = \beta E_t^i g_{p,t}^{j,\text{next year}} + \lambda^j + \lambda_t^i + \epsilon_t^{i,j}, \tag{2}$$

where $A_t^{i,j}$ are the total assets managed by fund j in month t, λ^j are fund fixed effects, λ_t^i are investor-time fixed effects, and $\epsilon_t^{i,j}$ is an error term.

The share of investor assets allocated to mutual fund j can be written $a_t^{i,j} = \frac{A_t^{i,j}}{\Omega_t^i}$, where $A_t^{i,j}$ is the total investor allocation to fund j. Our regression has investor-time fixed effects, so the above regression is equivalent to regressing $\ln\left(a_t^{i,j}\right)$ on the investor expectations and the fixed effects, where the investor's total assets are absorbed in the investor-time fixed effects. This specification helps us to estimate the impact of the expectations on the investor's allocation to the mutual fund even though we do not observe the investor's total wealth.

The use of investor-time fixed effects has many other advantages. It captures all the unobserved developments at the investor level that could drive the investor's allocation to the mutual fund. Among those are the global or investor-specific "funding shocks" that have been identified in the literature and could be correlated with expectations. They also include global or investor-specific expectation shocks that could, for instance, lead the investor to reallocate its wealth away from mutual equity funds and into bonds or cash. Finally, the fund fixed effects captures the investor-specific preference for a given fund. The identification of the role of expectations for investment into a fund comes from the relative evolution of the investor's expectation across funds (for example, if an investor's expectation about an Asian fund improve relative to a Latin American fund, then expectations matter if we observe an increase in the assets managed by the Asian fund increase relative to the Latin American fund).

Table 1, column (1) reports the results for Equation (2). Investor expectations of future GDP growth are positively associated with the flows allocated to mutual funds. Investor expectations impact mutual fund flows in an economically meaningful way. An increase in the aggregate weighted expected GDP growth by one percentage point is associated with an increase in investor allocations to the fund of about 15 percent.

Note that here we do not control for any fund-level time-specific development. Importantly, the fund-specific expectations could be correlated with the fund-specific equity returns or equity price changes, as equity price changes and returns may be relevant signals used to form expectations. On the other hand, equity price changes generate valuation effects that may or may not be balanced by the fund. To address this issue, we compute a measure

	(1)	(2)	(3)	(4)
VARIABLES	$\log(A_t^{i,j})$	$\log(A_t^{i,j})$	$\log(A_t^{i,j})$	$\log(A_t^{i,j})$
$E_t^i(g_p^{j,\text{next year}})$	0.154***	0.182***		
	(0.027)	(0.031)		
$\bar{E}_t^i(g_p^{j,\text{next year}})$			0.217***	0.217***
			(0.039)	(0.040)
$\Gamma_t^{i,j}$			0.056	0.097***
			(0.039)	(0.033)
$\Gamma_{t-1}^{i,j}$				-0.053
				(0.039)
$\Delta\log(A_t^j)$		-0.036***	-0.043***	-0.042***
		(0.012)	(0.012)	(0.012)
$\Delta\log(A_{t-1}^j)$		-0.029***	-0.036***	-0.036***
		(0.011)	(0.011)	(0.011)
$\Delta\log(Q_t^{i,j})$		-0.137**	-0.122**	-0.124**
		(0.062)	(0.058)	(0.057)
$\Delta\log(Q_{t-1}^{i,j})$		-0.113	-0.105	-0.102
		(0.078)	(0.071)	(0.070)
Observations	6,440	5,858	5,858	5,853
R-squared	0.833	0.841	0.843	0.843
Fund FE	Yes	Yes	Yes	Yes
Investor-time FE	Yes	Yes	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 1: Mutual Fund Flows and Investor Expectations

Note: The dependent variable $A_t^{i,j}$ is the log of investor i's allocation to fund j in logs, measured as the total assets under management of fund j during month t. Standard errors are clustered at the investor-year level.

of equity price log-changes at the fund level, which we denote $\Delta\log(Q_t^j)$. We compute this variables using the log-changes in the MSCI indices at the country level and aggregating using the countries' past allocations in the fund, as in Equation (1). We also control for the total capital flows $\Delta\log(A_t^j)$ that are invested into the set of countries the fund invests in. This accounts for a potential reverse causality from capital flows to growth and growth expectations. This variable is computed using the total flows at the country level, which we aggregate using the fund's past cross-country allocations, and taking the log-change. These two variables are added to specification (2) and the results, which are barely changed, are reported in column (2).

Another important issue is that we do not observe the investor's expectations for all the countries in the fund's portfolio. To understand, note that the "true" aggregate expectation can be decomposed into an observable and an unobservable component:

$$E_t^i(g_{p,t}^{j,\text{next year}}) = \sum_{k \in K(i,j)} w_{k,t-1}^{i,j} E_t^i(g_{k,t}^{\text{next year}}) + \sum_{k \in \tilde{K}(i,j)} w_{k,t-1}^{i,j} E_t^i(g_{k,t}^{\text{next year}})$$

where $\tilde{K}(i,j)$ is the set of countries for which we do not observe expectations. The first term is the variable that we use as a proxy for the true aggregate expectation. The second term is an unobservable variable that will be captured in the error term. This will generate a positive missing variable bias if the observable and unobservable terms are positively correlated, which is the case when they are driven by common shocks.

To circumvent this issue, we use the granular component of the expectations. We thus define an average investor expectation and a granular component defined as the weighted average of the differences between the country-specific expectation and an unweighted mean, along the lines of Gabaix (2011) and Gabaix and Koijen (2021):

$$\Gamma_t^{i,j} = \sum_{k \in K(i,j)} w_{k,t-1}^{i,j} \left[E_t^i(g_{k,t}^{\text{next year}}) - \frac{1}{N^{i,j}} \sum_{k \in K(i,j)} E_t^i g_{k,t}^{\text{next year}} \right] = E_t^i g_{p,t}^{j,\text{next year}} - \bar{E}_t^i g_{p,t}^{j,\text{next year}}. \tag{3}$$

with

$$\bar{E}_t^i g_{p,t}^{j,\text{next year}} = \left(\sum_{k \in K(i,j)} w_{k,t-1}^{i,j} \right) \frac{1}{N^{i,j}} \sum_{k \in K(i,j)} E_t^i g_{k,t}^{\text{next year}}, \tag{4}$$

where $N^{i,j}$ is the number of elements in $K(i,j)$. If we assume that expectations are equal to a common driver and an idiosyncratic one, then the granular component is orthogonal to the common drivers of expectations, and it is thus immune to the positive bias. Note that here, contrary to Gabaix (2011) and Gabaix and Koijen (2021), the unweighted average needs to be rescaled.

In Table 1, column (3) reports the following regression that decomposes the aggregate expectation into their simple average and the granular term:

$$\ln\left(A_t^{i,j}\right) = \beta_1 \bar{E}_t^i g_p^{j,\text{next year}} + \beta_2 \Gamma_t^{i,j} + \beta_3 \Delta(\log(A_t^j)) + \beta_4 \Delta(\log(Q_t^j)) + \lambda^j + \lambda_t^i + \epsilon_t^{i,j}. \tag{5}$$

Both the simple average and the granular component are positive and significant. Interestingly, the coefficient of the granular component is lower than the coefficient of the aggregate expectation while the coefficient of the simple average is higher, which confirms the presence of a positive missing variable bias. It is also now only significant at the 16% level.

Column (4) reports the same regression but adds the first lag of the granular component. The contemporaneous response is larger and significant, while the lagged response is insignificant. In this last specification, an increase in the aggregate weighted expected GDP growth by one percentage point is associated with an increase in investor allocations to the fund of about 10 percent.

3.2 Investor expectations and country allocations

Next, we test the relationship between investor expectations and the country allocation of the mutual funds. We run the following regression at the fund-country level,

$$\log\left(w_{k,t}^{i,j}\right) = \beta E_t^i g_k^{\text{next year}} + \lambda_k^j + \lambda_{k,t} + \lambda_t^i + \epsilon_{k,t}^{i,j}., \tag{6}$$

where $w_{k,t}^{i,j}$ is fund j's allocation to country k in percent of assets under management of investor i and $E_t^i x_{k,t+1}$ is investor i's expectations for future GDP growth in percent for country k. Fund-country fixed effects λ_k^j capture heterogeneity in the funds' preferences for countries. Investor-time fixed effects take into account global and investor-specific time-varying outside investment opportunities as well as global and investor-specific funding shocks.

Importantly, country-time fixed effects take into account country-specific developments that simultaneously drive the country's supply of capital (and thus allocations $w_{k,t}^{i,j}$) and expectations, such as country growth, changes in local equity prices and monetary policy. They also capture reverse causality from capital flows to expectations, as capital flow surges may temporarily stimulate growth and growth expectations, or, on the opposite, increase the risks of a downturn. The coefficient β is identified through the time variation in the idiosyncratic differences in investor expectations regarding a country relative to other countries.

Results of regression (6) are shown in Table 2. In column (1), the response of mutual funds to the investor forecasts is significant but relatively small: a 1 percentage point rise in the investor's growth forecast regarding a country increases the share of wealth invested in that country by about 6% (so a country with an initial 10% share will benefit from a 0.6 percentage point increase). This is almost twice as low as the the response of flows into the funds reported in the last column of Table 1. For a smaller subset of global emerging market (GEM) funds, the response of the fund portfolio allocation to investor forecast is more economically significant: one percentage point increase in the GDP growth forecast is associated with an 13% increase in the country allocation, as shown in column (2).

Interestingly, when introducing the lagged expectations $E^i_{t-1} g_k^{\text{next year}}$ in columns (3) and (4), it appears that allocations do not actually respond contemporaneously to the investor's expectations as only the lagged expectation is significant, except for GEM funds.

	(1)	(2)	(3)	(4)
	$\log(w^{i,j}_{k,t})$	$\log(w^{i,j}_{k,t})$	$\log(w^{i,j}_{k,t})$	$\log(w^{i,j}_{k,t})$
VARIABLES	All funds	GEM funds	All funds	GEM funds
$E^i_t(g_k^{\text{next year}})$	0.059**	0.128***	0.020	0.095***
	(0.024)	(0.035)	(0.024)	(0.032)
$E^i_{t-1}(g_k^{\text{next year}})$			0.052**	0.091***
			(0.022)	(0.033)
Observations	36,007	8,276	29,998	6,939
R-squared	0.944	0.937	0.946	0.947
Country-fund FE	Yes	Yes	Yes	Yes
Country-time FE	Yes	Yes	Yes	Yes
Fund-time FE	Yes	Yes	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: Mutual Fund Allocations, Investor Expectations

Note: The dependent variable is the log of $w^{i,j}_{k,t}$, the share of fund j's assets under management that is allocated to country k in month t. Standard errors are clustered a the investor-year and country-year levels.

All in all, this empirical section has established that, even though flows into funds respond to investors' expectations, the funds' cross-country allocations remain relatively sticky.

4 Model

Motivated by the empirical evidence, the model presented in this section serves several purposes. First, it helps us understand how sticky portfolios affect the relation between expectations and capital flows at the country and mutual fund level. Second, it enables us to discuss the aggregate consequences of the friction. Third, it will help us map relevant model parameters to the data and quantify these aggregate consequences. We first consider a simpler version of the model where investors are paired with only one mutual fund, then we analyze an extension where investors are paired with several mutual funds.

We outline a simple, two-period model of portfolio choice. There are M investors indexed by $i = 1, .., M$. Each investor i is paired with one equity mutual fund. In the first period, investors choose between investing in a safe asset and in the equity mutual fund, and equity mutual funds invest in the equity markets of N countries, indexed by $k = 1, .., N$. In the second period, portfolio returns are realized. Equity investments pay a stochastic dividend that is specific to the country. A fund's return thus depends on the country weights in the funds' portfolio. Investors and mutual funds maximize the same objective, which is the investors' utility.

Information and frictions in delegation are modeled as follows. In the first period, investors and mutual funds obtain information on the fundamental driving the stochastic dividend. We assume that investors and mutual funds share the same information and the same expectation formation process. Investors choose their portfolio allocation between the safe asset and the equity mutual fund conditional on that information, but cannot decide the mutual funds' allocation between countries. We assume that a mutual fund is able to update its allocation rule conditional on the new information only with probability $p \leq 1$. With a probability $1 - p$, the fund does not update its portfolio.

Investors and mutual funds are denoted with superscript i, and countries with subscript k.

4.1 Country returns and expectations

Equity held in country $k = 1, .., N$ pays a stochastic dividend in period $t+1$, $d_{k,t+1}$. Country k equity is traded at price $q_{k,t}$ in period t, so that the return is $R_{k,t+1} = \frac{D_{k,t+1}}{Q_{k,t}}$, where $Q_{k,t}$ is the price of a share in country k on period t and $D_{k,t+1}$ is the associated dividend on period $t + 1$. We log-linearize the dividends and the share price around the world' averages D and Q, and we normalize $D/Q = 1$, so that the returns have a simple linear form:

$$R_{k,t+1} = \frac{D_{k,t+1}}{Q_{k,t}} = 1 + d_{k,t+1} - q_{k,t} \tag{7}$$

with $d_{k,t+1} = \log(D_{k,t+1}) - \log(D)$ and $q_{k,t} = \log(Q_{k,t}) - \log(Q)$.

We denote the vector of log-linearized dividends by $d_{t+1} = (d_{1,t+1}, .., d_{k,t+1}, .., d_{N,t+1})'$, the vector of log-linearized asset prices by $q_t = (q_{1,t}, .., q_{k,t}, .., q_{N,t})'$ and the vector of returns by $R_{t+1} = (R_{1,t+1}, .., R_{k,t+1}, .., R_{N,t+1})'$. We assume that the log-linearized dividends are exogenous and follow a Gaussian distribution: $d_{t+1} \sim \mathcal{N}(d, \Sigma)$, where $d = (d_1, .., d_k, .., d_N)'$ is the vector of the unconditional mean and Σ is the matrix of variance-covariance.

An investor-mutual fund pair $i = 1, .., M$ shares the same information on the fundamental

d_{t+1}. In period t, we distinguish between the beginning-of-period information of investor-fund pair i, $\bar{\mathcal{I}}^i$, and their end-of period information $\mathcal{I}_t^i$. We assume that $q_t \in \mathcal{I}_t^i$, since q_t is an observable equilibrium variable. We denote by $\bar{E}^i(.) = E(.|\bar{\mathcal{I}}^i)$ the expectations conditional on $\bar{\mathcal{I}}^i$, the beginning-of-period information, and by $E_t^i(.) = E(.|\mathcal{I}_t^i)$ the expectations conditional on the end-of-period information. We have a relationship between the expected returns and the expected fundamentals d_{t+1}

$$\bar{E}_t^i(R_{t+1}) = 1 + \bar{E}_t^i(d_{t+1}) - \bar{E}_t^i(q_{t+1}), \qquad E_t^i(R_{t+1}) = 1 + E_t^i(d_{t+1}) - q_{t+1} \tag{8}$$

We denote by $\bar{V}(.) = V(.|\bar{\mathcal{I}}^i)$ the variance conditional on $\bar{\mathcal{I}}^i$, and by $V(.) = V(.|\mathcal{I}_t^i)$ the variance conditional on $\mathcal{I}_t^i$. We denote by $\bar{V}^R$ and V^R the conditional variances of returns:

$$\bar{V}^R = \bar{V}(R_{t+1}), \qquad V^R = V(R_{t+1}) \tag{9}$$

It will be useful to make the following assumption on the structure of learning:

Assumption 4.1 $\bar{V}^R - V^R << V^R$.

This assumption states that the change in the conditional variance of returns between the beginning of period and the end of period is small compared to the conditional variance at the end of period.

4.2 Investors

Investor i enters period t with initial wealth Ω_t^i and invests a share a_t^i in equity fund i, which invests in countries $k = 1, .., N$, and a share $1 - a_t^i$ in a period bond. The decisions of the investor are taken after observing the new information $\mathcal{I}_t^i$, but before observing the country allocation of the fund.

In period $t + 1$, portfolio returns are realized and the investor consumes all remaining terminal wealth defined as

$$\Omega_{t+1}^i = \left[R_{p,t+1}^i a_t^i + r(1 - a_t^i)\right] \Omega_t^i, \tag{10}$$

where the equity fund portfolio return $R_{p,t+1}^i$ is defined as

$$R_{p,t+1}^i = \sum_{k=1}^{N} w_{k,t}^i R_{k,t+1} = w_t^{i'} R_{t+1} \tag{11}$$

where the real gross return on the safe asset is r, the return on country k's equity is $R_{k,t+1}$

and $w^i_{k,t}$ is the share of mutual fund i's portfolio that are invested in country k. The vector $w^i_t = (w^i_{1,t}, .., w^i_{k,t}, .., w^i_{N,t})'$ collects the country shares. Investors take the portfolio return as given. As we will see below, the country shares depend on whether the mutual fund updates its portfolio or not, which the investor does not know when deciding a^i_t.

Investors have mean-variance preferences and choose the investment share a^i_t to maximize the mean-variance utility of one unit of wealth,

$$U^i_{t+1} = E^i_t \left[R^i_{p,t+1} a^i_t + r(1 - a^i_t) \right] - \frac{\gamma}{2} V \left[R^i_{p,t+1} a^i_t + r(1 - a^i_t) \right], \tag{12}$$

where $E^i_t(.)$ and $V(.)$ are defined as the expectation and variance conditional on $\mathcal{I}^i_t$ and q_t as stated above, subject to the wealth accumulation equation (10) and the aggregate equity return (11), and taking the distribution of returns R and of portfolio shares w^i_t as given.

The optimal share of investment in equity must then satisfies

$$a^i_t = \frac{E^i_t(R^i_{p,t+1}) - r}{\gamma V(R^i_{p,t+1})} \tag{13}$$

4.3 Mutual Funds

After investor i has decided her investment $a^i_t \Omega^i_t$ in fund i, the fund allocates $a^i_t \Omega^i_t$ across the different countries as follows.

At the beginning of period, the fund is endowed with information $\bar{\mathcal{I}}^i$ and sets the default country shares $\bar{w}^i$. The fund chooses the default country allocation $\bar{w}^i$ by maximizing the same objective (12) as the investor, but conditional on the beginning-of-period information $\bar{\mathcal{I}}^i_t$, subject to Equations (10), (11), $\sum_{k=1}^N \bar{w}^i_k = 1$ and taking the distribution of returns R as given. We can show that $\bar{w}^i$ satisfies, for all $k = 1, .., N$

$$\bar{E}^i(R_{t+1}) - \bar{E}^i(R_{k,t+1}) = \gamma(\bar{V}^R - \bar{V}^R_k)\bar{w}^i \bar{E}^i(a^i_t) \tag{14}$$

where $\bar{E}^i(a^i_t)$ is the expected investment share in fund i, and where each line of $\bar{V}^R_k$ is equal to $\bar{v}^R_k$, the k^{th} line of $\bar{V}^R$.

With probability $1-p$, the fund allocates the resources received from the investor across countries following the default portfolio shares $\bar{w}^i$. With probability p, the fund updates its portfolio after observing $\mathcal{I}^i_t$, i.e. the same information as the investor.The fund then chooses the country allocation w^{i*}_t by maximizing the same objective (12) as the investor, conditional on $\mathcal{I}^i_t$, subject to Equations (10) and (11), $\sum_{k=1}^N w^{i*}_{k,t} = 1$ and taking the distribution of returns R as given. We can show that w_t^{i*} satisfies, for all $k = 1, .., N$

$$E_t^i(R_{t+1}) - E_t^i(R_{k,t+1}) = \gamma(V^R - V_k^R)w_t^{i*}a_t^i \tag{15}$$

where each line of V_k^R is equal to v_k^R, the k^{th} line of V^R.

4.4 Asset demand

Combining Equations (11), (13), (14) and (15), we can describe the asset demand for each country $k = 1, .., N$. We define the expected share of investment to country k, conditional on $\mathcal{I}_t^i$, as $a_{k,t}^i = \tilde{w}_{k,t}^i a_t^i$, where $\tilde{w}_{k,t}^i = pw_{k,t}^{i*} + (1-p)\bar{w}_k^i$ is the expected fund allocation to country k. These flows depend both on the share allocated to the fund a_t^i and on the expected fund country allocation $\tilde{w}_{k,t}^i$.

The following lemma shows that two types of spillovers arise, portfolio reallocation spillovers, and co-ownership spillovers. The latter appear only in the presence of the portfolio friction.

Lemma 4.1 (Spillovers) *In the presence of a portfolio friction (if $p < 1$), and if Assumption 4.1 is satisfied, the final allocation to country k from investor i, $a_{k,t}^i = w_{k,t}^i a_t^i$, is given by:*

$$\begin{aligned} a_{k,t}^i =& p\frac{E_t^i(R_{k,t+1}) - r}{\gamma V_k^i} \\ &- p\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1})}{V_k^i}a_t^i \\ &+ (1-p)\bar{w}_k^i a_t^i \end{aligned} \tag{16}$$

where $V_k^i = V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)$, a_t^i is given by Equation (13) and $Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)$ is the covariance between the return of the country k asset and the return of the portfolio that excludes k, $R_{p,k^-,t+1}^i = \sum_{j,j\neq k}^N w_{j,t}^i R_{j,t+1}/(1 - w_{k,t}^i)$.

Proof. See proof in Appendix B.1. ■

Consider first the result in the absence of return correlation, ($Cov(R_{k,t+1}, R_{p,k^-,t+1}) = 0$) and without portfolio friction ($p = 1$). In that case, investment in country k is only affected by the expectations about country k's excess return, and is not affected by the total flows to the fund a_t^i. This implies that investment in country k is independent from the expectations about other countries in the portfolio.

Concretely, if the investor receives new information so that she expects higher returns in country j, she will increase her allocation to the equity fund a_t^i. If the equity fund does not update its information, then these extra resources will be channeled to the countries according to previous information, generating spillovers to country k. But if the equity fund updates its portfolio, then it will increase the share that goes to country j. This portfolio reallocation offsets the mechanical flow to country k due to the increased investment in the fund.

But in general, some spillovers arise through portfolio reallocation. They appear in the second term in Equation (16), which depends on the covariance between the return in country k and the return on the rest of the portfolio ($Cov(R_{k,t+1}, R_{p,k^-,t+1})$), and, through a_t^i, on the expectation on the overall portfolio return $E_t^i(R_{p,t+1})$. Suppose that the covariance is positive. Higher expectations about country j will generate a negative spillover on investment in country k, because country k is a close substitute to the rest of the portfolio. In that case, the portfolio reallocation spillovers are negative.

Consider now the last term in Equation (16). Take the same example as above, where the investor receives good news about country j. With the information friction ($p < 1$), the mutual fund does not adjust its information with some positive probability ($1 - p > 0$), which implies that some of the funds destined to j end up in k. This spillover is positive whenever the "default" portfolio share $\bar{w}_k^i$ is positive. Since funds typically don't take short positions, these co-ownership spillovers are positive.

Finally, when the mutual fund's portfolio is sticky ($p < 1$), the capital allocated to country k is less elastic to the updated expectation on k's return $E_t^i(R_{k,t+1})$. This is because some funds destined to country k are channeled to other countries that are part of the portfolio if the portfolio shares are not adjusted.

If we take into account the fund's optimal setting of the default portfolio shares, we obtain the following capital flows as a function of expectations:

Proposition 4.1 *We assume that Assumption 4.1 is satisfied. In that case, Equation* (16) *can be written as:*

$$\begin{aligned} a_{k,t}^i =& p\frac{E_t^i(R_{k,t+1}) - r}{\gamma V_k^i} \\ &+ (1-p)\frac{\bar{E}^i(R_{k,t+1}) - r}{\gamma V_k^i \bar{E}^i(a_t^i)} a_t^i \\ &- \frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)}{V_k^i} a_t^i \end{aligned} \tag{17}$$

with $V_p^i = V(R_{p,t+1}^i)$.

Proof. See proof in Appendix B.2. ■

This proposition shows that the portfolio friction does not affect the portfolio reallocation spillovers, as we can see that the third term of Equation (17) does not depend on p. Indeed, these spillovers arise automatically from the "fixed" part of the portfolio share, which does not depend on expectations. The co-ownership spillovers arise from the ex ante excess return expectation for country k, $\bar{E}^i(R_{k,t+1}) - r$, which defines the part of the portfolio share of k that is truly "sticky", i.e. the part that would be adjusted in the absence of portfolio stickiness.[6]

4.5 Aggregate capital flows

Consider total capital flows to country $k = 1, .., N$. These correspond to the sum over all investor-mutual fund pairs $i = 1, .., M$: $A_{k,t} = \sum_{i=1}^{M} A_{k,t}^i$, where $A_{k,t}^i = a_{k,t}^i \Omega_t^i$ is the total flow from investor-fund i to country k. We will focus on $a_{k,t} = A_{k,t}/\Omega_t$, the share of total wealth $\Omega_t = \sum_{i=1}^{M} \Omega_t^i$ that goes to country k. We have

$$a_{k,t} = \sum_{i=1}^{M} \frac{\Omega_t^i}{\Omega_t} a_{k,t}^i \tag{18}$$

The share of global wealth that is invested in country k is an average of the individual investor shares, weighted by the investor-fund contribution to total wealth.

We now focus on the unexpected investment share to k, scaled by the expected share, and show that it relates to the investor-level unexpected investment shares to k:

$$\frac{a_{k,t} - \bar{E}(a_{k,t})}{\bar{E}(a_{k,t})} = \sum_{i=1}^{M} \sigma_{k,t}^i \frac{a_{k,t}^i - \bar{E}^i(a_{k,t}^i)}{\bar{E}^i(a_{k,t}^i)} \tag{19}$$

where $\sigma_{k,t}^i = \bar{E}^i(a_{k,t}^i)\Omega_t^i / \sum_{i=1}^{M} \bar{E}^i(a_{k,t}^i)\Omega_t^i$ is the share of investor-fund in the total flows to country k. We used the fact that, because the Ω_t^is are known in the beginning of period t, $\bar{E}(a_{k,t}) = \sum_{i=1}^{M} \frac{\Omega_t^i}{\Omega_t} \bar{E}^i(a_{k,t}^i)$. As a result, surprises in capital flows are due to surprises in return expectations at the investor level, not to surprises in wealth (funding), which has been the focus of the literature thus far. These surprises at the investor level weigh more if the investor's average flows to k are relatively large.

According to Proposition 4.1, the share of wealth invested to country k by investor-fund i $a_{k,t}^i$ can be decomposed into a term that depends on the expectation on the country-k return and a term that depends on the expectation on the whole portfolio. We can then write the

[6] Here, Assumption 4.1 ensures that the "fixed" part of the portfolio shares, which depends on the ratio of the conditional covariance to the variance, is invariant whether the fund updates its shares or not and that the default shares are not significantly affected by any precautionary behavior.

surprise aggregate share as follows:

$$\frac{a_{k,t} - \bar{E}(a_{k,t})}{\bar{E}(a_{k,t})} = \sum_{i=1}^{M} \sigma_t^i \left(\beta_k^i [E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1})] + \delta_k^i [E_t^i(R_{p,t+1}^i) - \bar{E}^i(R_{p,t+1}^i)]\right) \quad (20)$$

they only depend on the revisions in the country-specific and portfolio-specific return expectations, with β_k^i and δ_k^i the elasticities of capital flows to the country-specific expectations and to the fund-specific expectations. According to our model, these elasticities are

$$\begin{aligned} \beta_k^i &= p \frac{1}{\gamma V_k^i \bar{E}^i(a_{k,t}^i)} \\ \delta_k^i &= (1-p) \frac{\left[\bar{E}^i(R_{k,t+1}) - r\right]}{\gamma^2 V_k^i V_p^i \bar{E}^i(a_t^i) \bar{E}^i(a_{k,t}^i)} - \frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)}{\gamma V_k^i V_p^i \bar{E}^i(a_{k,t}^i)}. \end{aligned} \quad (21)$$

Note that δ_k^i, the elasticity to the fund-specific expectations, can be decomposed into two terms:

$$\delta_k^i = \eta_k^i - \frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)}{V_p^i} \phi_k^i \quad (22)$$

with $\eta_k^i = (1-p)\frac{\bar{E}^i(R_{k,t+1}) - r}{\gamma^2 V_k^i V_p^i \bar{E}^i(a_t^i)\bar{E}^i(a_{k,t}^i)}$ and $\phi_k^i = \frac{1}{\gamma V_k^i \bar{E}^i(a_{k,t}^i)}$. The first term, η_k^i, captures the co-ownership spillovers while the second term captures the portfolio reallocation spillovers.

Note that we can write all end-of-period revisions in expectations as the sum of a local component $l_{k,t}^i$ and a global component W_t^i:

$$E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1}) = l_{k,t}^i + W_t^i \quad (23)$$

where W_t^i is the simple average of the country expectations of investor i: $W_t^i = \frac{1}{N}\sum_{k=1}^{N}[E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1})]$ and $l_{k,t}^i$ is a country-specific residual: $l_{k,t}^i = E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1}) - X_{W,t}^i$. Note that, by construction, $\sum_{k=1}^{N} l_{k,t}^i = 0$. Therefore, the portfolio return expectations can be decomposed into a global and a "granular" component:

$$E_t^i(R_{p,t+1}) - \bar{E}^i(R_{p,t+1}) = \Gamma_t^i + W_t^i \quad (24)$$

where the granular component Γ_t^i is by construction the weighted average of the local components:

$$\Gamma_t^i = \sum_{k=1}^{N} \left(w_{k,t}^i - \frac{1}{N}\right) [E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1})] = \sum_{k=1}^{N} \tilde{w}_{k,t}^i l_{k,t}^i = \tilde{w}_t^{i'} l_t^i \quad (25)$$

where $l_t^i = (l_{1,t}^i, .., l_{k,t}^i, .., l_{N,t}^i)'$ is the vector of local components.

The surprises in capital flows can then be decomposed as follows:

$$\frac{a_{k,t} - \bar{E}(a_{k,t})}{\bar{E}(a_{k,t})} = \sum_{i=1}^{M} \sigma_{k,t}^i \beta_k^i l_{k,t}^i + \sum_{i=1}^{M} \sigma_{k,t}^i (\beta_k^i + \delta_k^i) W_t^i + \sum_{i=1}^{M} \sigma_{k,t}^i \delta^i \Gamma_t^i \tag{26}$$

In what follows, we compare the effective equilibrium capital flows to the "frictionless" capital flows that would hold in the absence of portfolio friction. It is useful to make the following assumption:

Assumption 4.2 *For all $i = 1, .., M$, $\bar{E}^i(R_{k,t+1}) = \bar{E}^i(R_{k',t+1})$ for all $k' \neq k$.*

We then derive the following corollary:

Corollary 4.1 *β_k^i is decreasing in $1-p$. δ_k^i is increasing in $1-p$ and is positive for a large $1-p$. Under Assumption 4.2 $\beta_k^i + \delta_k^i$ is independent of p. Additionally, $\beta_k^i/\eta_k^i = p/(1-p)$.*

When $p < 1$, the β_k^i terms, i.e. the reactions of capital flows to the investors' country expectations, are lower than what they would be in the optimum (with $p = 1$), which means that the response of capital flows to the country-k specific expectations is too sticky as compared to the frictionless benchmark.

It is different for the granular term. Indeed, the response to capital flows responds more positively to the granular component when the portfolio becomes more sticky (when p decreases). If $1-p$ is large, the co-ownership spillovers dominate the portfolio reallocation and δ_k^i becomes positive. In that case, a larger $1-p$ increases δ_k^i, and the granular component generates extra capital flow volatility. Therefore, as p declines (as portfolios becomes more sticky), the contribution of the country component of expectations to the country capital flows declines, while the contribution of the granular component increases.

Interestingly, under symmetric ex ante expectations, the reaction to the global component of expectations, $\beta_k^i + \delta_k^i$ does not depend on the friction and is equal to the optimal response. As a result, the correlation between capital flows across countries increases when p declines, and this is due to the granular component of expectations, and not to the global component of expectations.

The last result states that the ratio of β_k^i over η_k^i, that is, the elasticity to the country expectations over the co-ownership spillover coefficient, provides an approximation for the strength of the friction.

4.6 Extension with multiple funds per investor

We consider here the more realistic case where a given investor i is associated with more than one fund. We therefore denote a fund by the index $j = 1, .., J(i)$ to distinguish it from the investor index i. Each fund potentially invests in a different set of countries. We denote by $\mathcal{S}(i,j)$ the set of countries in which fund j managed by investor i invests.

Now the investor budget constraint is

$$\Omega_{t+1}^{i} = \left[\mathcal{R}_{p,t+1}^{i} \left(\sum_{j=1}^{J(i)} a_t^{i,j} \right) + r \left(1 - \sum_{j=1}^{J(i)} a_t^{i,j} \right) \right] \Omega_t^{i}, \tag{27}$$

where $a_t^{i,j}$ is the share of investor i wealth invested in mutual fund j and $\mathcal{R}_{p,t+1}^{i}$ is the return on the total equity fund investments of investor i:

$$\mathcal{R}_{p,t+1}^{i} = \sum_{j=1}^{J(i)} \frac{a_t^{i,j}}{\sum_{j=1}^{J(i)} a_t^{i,j}} R_{p,t+1}^{i,j} \tag{28}$$

with $R_{p,t+1}^{i,j}$ the aggregate return on mutual fund j's portfolio. We assume as before that each fund adjusts her portfolio with probability $p \leq 1$. The return on the portfolio of fund j managed by investor i is therefore equal to

$$\begin{aligned} R_{p,t+1}^{i,j} &= \sum_{k \in \mathcal{S}(i,j)} w_{k,t}^{i,j*} R_{k,t+1} && \text{if the fund updates} \\ &= \sum_{k \in \mathcal{S}(i,j)}^{N} \bar{w}_k^{i,j} R_{k,t+1} && \text{if the fund does not} \end{aligned} \tag{29}$$

Updating funds choose $w_{k,t}^{i,j*}$ in order to maximize the investor's utility (12) subject to the wealth accumulation equation (27) and the aggregate equity return (29), and conditional on the end-of-period information $\mathcal{I}_t^i$. The default portfolio shares $\bar{w}_k^{i,j}$ are set to maximize the utility of the investor conditional on the beginning-of-period information $\bar{\mathcal{I}}^i$.

The other assumptions remain unchanged. In particular, we still assume that the investor and the funds share the same information, so information variables remain indexed by i only. We further assume that Assumption 4.1 is satisfied, and that $\bar{E}^i(a_t^{i,j}) \simeq \bar{a}^{i,j}$ where we define $\bar{a}^{i,j}$ as the share of investor i' wealth invested in fund i that would hold under the beginning-of-period information. We derive the equivalent of Lemma 4.1 and Proposition 4.1 in Appendix A. We focus here on the aggregate implications of the portfolio friction.

Regarding the revisions in expectations, we write all end-of-period revisions in expectations as the sum of a global component $W_t^i = \frac{1}{N}\sum_{k=1}^{N}[E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1})]$ and a local component $l_{k,t} = E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1}) - W_t^i$, as before, so that the portfolio return expectations can be decomposed into a global and a "granular" component at the fund level:

$$E_t^i(R_{p,t+1}^{i,j}) - \bar{E}^i(R_{p,t+1}^{i,j}) = \Gamma_t^{i,j} + W_t^i \tag{30}$$

where the fund-specific granular component $\Gamma_t^{i,j}$ is again, by construction, the weighted average of the local components:

$$\Gamma_t^{i,j} = \sum_{k=1}^{N}\left(\tilde{w}_{k,t}^{i,j} - \frac{1}{N}\right)[E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1})] = \sum_{k\in\mathcal{S}(i,j)} \tilde{w}_{k,t}^{i,j} l_{k,t}^i = \tilde{w}_t^{i,j\prime} l_t^i, \tag{31}$$

but also at the investor level:

$$E_t^i(\mathcal{R}_{p,t+1}^i) - \bar{E}^i(\mathcal{R}_{p,t+1}^i) = \Gamma_t^i + W_t^i \tag{32}$$

where the granular component Γ_t^i is described by

$$\begin{aligned}\Gamma_t^i &= \sum_{k=1}^{N}\left(\tilde{w}_{k,t}^{i} - \frac{1}{N}\right)[E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1})] \\ &= \tilde{w}_t^{i\prime} l_t^i \end{aligned}\tag{33}$$

where $\tilde{w}_{k,t}^i$ is the share of investor i's wealth that in invested in country k and $\tilde{w}_t^i$ is a vector that collects these shares.

The surprises in capital flows can be decomposed as follows:

$$\begin{aligned}\frac{a_{k,t} - \bar{E}(a_{k,t})}{\bar{E}(a_{k,t})} &= \sum_{i=1}^{M}\sum_{j=1}^{J(i)} \sigma_{k,t}^{i,j}\frac{a_{k,t}^{i,j} - \bar{E}^i(a_{k,t}^{i,j})}{\bar{E}^i(a_{k,t}^{i,j})} \\ \sum_{i=1}^{M} l_{k,t}^i \sum_{j=1}^{J(i)} \sigma_{k,t}^{i,j}\beta_k^{i,j} &+ \sum_{i=1}^{M} W_t^i \sum_{j=1}^{J(i)} \sigma_{k,t}^{i,j}(\beta_k^{i,j} + \delta_k^{i,j} + \theta_k^{i,j}) + \sum_{i=1}^{M}\sum_{j=1}^{J(i)} \Gamma_t^{i,j}\sigma_{k,t}^{i,j}\delta_k^{i,j} + \sum_{i=1}^{M}\Gamma_t^i \sum_{j=1}^{J(i)} \sigma_{k,t}^{i,j}\theta_k^{i,j}\end{aligned}\tag{34}$$

where $a_{k,t}^{i,j} = w_{k,t}^{i,j} a_t^{i,j}$ is the share of investor's i' wealth that is invested in country k through fund j, $\sigma_{k,t}^{i,j} = \bar{E}^i(a_{k,t}^{i,j})\Omega_t^i / \sum_{i=1}^{M}\Omega_t^i \sum_{j=1}^{J}(i)\bar{E}^i(a_{k,t}^{i,j})$ is the share of fund j managed by investor i in the total flows to country k, and $\beta_k^{i,j}$, $\delta_k^{i,j}$, $\theta_k^{i,j}$ are the elasticities of capital flows to the country-specific expectations, the fund-specific expectations and to the investor-specific expectations (that is, expectations on the returns of the whole investor's portfolio).

As compared to the simple case, there are some additional spillovers from the investor-wide expectations, that is, from the investor's expectations on its whole portfolio. These spillovers are governed by the $\theta_k^{i,j}$ parameters. These expectations also include a common component and a granular one. Besides, now $\delta_k^{i,j}$ is written as follows:

$$\delta_k^{i,j} = \eta_k^{i,j} - \left(\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j}) - Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_p^{i,j}} \right) \phi_k^{i,j} \tag{35}$$

where $\eta_k^{i,j}$ is proportional to $1-p$ and $phi_k^{i,j}$ is invariant in p. See Appendix A for a precise definition of these parameters and of $\eta_k^{i,j}$, $phi_k^{i,j}$, $\beta_k^{i,j}$ and $\theta_k^{i,j}$. The parameter $\eta_k^{i,j}$ represents the same co-ownership spillovers as in the simple case. The second term is close to the portfolio reallocation spillovers that we find in the second line of Equation (16), with the nuance that now this term is positively influenced by the covariance between the return in country k and the overall portfolio that excludes fund j. This effect comes from the fact that, for a given total allocation of investor i to equity funds, a higher allocation to fund j implies that investment in the other funds is less attractive. If returns in country k are positively correlated with the returns in these other funds, then some capital is reallocated to country k as k would be a relatively more profitable close substitute to these other funds.

We compare again the effective equilibrium capital flows to the "frictionless" capital flows that would hold in the absence of portfolio friction. Before that, it is useful to the define the following assumption:

Assumption 4.3 *For all $i = 1, .., M$ and $j = 1, .., J(i)$, $\bar{E}^i(R_{k,t+1}) = \bar{E}^i(R_{k',t+1})$ for all $k' \neq k$, and $Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j}) = Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,j^-,t+1}^{i,j})$.*

We then derive the following corollary:

Corollary 4.2 *$\beta_k^{i,j}$ is decreasing in $1-p$. δ_k^i is increasing in $1-p$ and is positive for a large p. Under Assumption 4.3, $\beta_k^{i,j} + \delta_k^{i,j} + \theta_k^{i,j}$ and $\theta_k^{i,j}$ are independent of p. Besides, $\beta_k^{i,j}/\eta_k^{i,j} = p/(1-p)$.*

The results are similar to the simple case with a single fund per investor. The response of capital flows to the country-k specific expectations becomes stickier as p declines. Under symmetric ex ante expectations and symmetric covariances, the reaction to the global component of expectations is equal to the optimal response. Interestingly, the response to the investor-level granular component Γ_t^i is also independent of p.

It is different again for the fund-level granular term $\Gamma_t^{i,j}$. Indeed, the response to capital flows responds more positively to the granular component when the portfolio becomes more sticky. If the co-ownership spillovers dominate the portfolio reallocation spillovers, only the granular component can generate extra capital flow volatility. Besides, as p declines, the cross-country correlation in capital flows increases due to that component.

4.7 Asset producers and general equilibrium

We now introduce asset producers in order to close the model. We use the version of the model with multiple funds per investors. In general equilibrium, we can determine how the investors' expectations affect asset prices. We can thus analyze the role of the portfolio frictions in generating both correlation in capital flows and in asset prices.

4.7.1 Asset producers

Each country $k = 1, .., N$ is endowed with $\bar{K}_k$ units of asset that can be supplied to investors. Asset producers supply additional asset $K_{k,t} - \bar{K}_k$ with a quadratic cost. The profits of asset producers in country $k = 1, .., N$ are:

$$\Pi_t = (Q_{k,t} - 1)(K_{k,t} - \bar{K}_k) - \frac{\phi}{2}\frac{1}{\bar{K}_k}(K_{k,t} - \bar{K}_k)^2 \tag{36}$$

Profit maximization by the asset producers yields the following equation, for each $k = 1, .., N$:

$$q_{k,t} = \phi\frac{1}{\bar{K}_k}(K_{k,t} - \bar{K}_k) + 1 \tag{37}$$

ϕ determines the elasticity of asset supply. $\phi = 0$ corresponds to a hyperelastic asset supply. The supply of assets becomes completely inelastic when ϕ goes to infinity.

In equilibrium, we must have that the demand for capital is equal to the supply, so that

$$K_{k,t} = A_{k,t} \tag{38}$$

It will be useful to make the following simplifying assumption:

Assumption 4.4 $\phi = 0$.

In the simpler case where $\phi = 0$, the supply of capital is hyperelastic, which implies that $Q_{k,t} = 1$. We will consider the more general case where $\phi > 0$ in an extension of the model. In that case, expectation shocks translate both into quantities (capital flows) and into prices. Under Assumption, 4.4, there are only consequences on quantities.

5 Identification

The purpose of this section is to identify the terms in Equation (34). For this purpose, we first focus on the β and δ coefficients, then on the θ coefficients.

5.1 A Mapping from Model to Data

The scaled surprise capital flow to country k by investor i through fund j can be expressed as:

$$\frac{a_{k,t}^{i,j} - \bar{E}^i(a_{k,t}^{i,j})}{\bar{E}^i(a_{k,t}^{i,j})} = \beta_k^{i,j}[E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1})] + \delta_k^{i,j}[E_t^i(R_{p,t+1}^{i,j}) - \bar{E}^i(R_{p,t+1}^{i,j})] + \theta_k^{i,j}[E_t^i(\mathcal{R}_{p,t+1}^i) - \bar{E}^i(\mathcal{R}_{p,t+1}^i)] \quad (39)$$

We want to identify β_k^i, δ_k^i and θ_k^i. We assume that these coefficients are homogenous across investors and countries, so that $\beta_k^i = \beta$, $\delta_k^i = \delta$ and $\theta_k^i = \theta$.

We first focus on the identification of β_k^i and δ_k^i. We approximate surprises in returns as follows

$$\begin{aligned} E_t^i(R_{k,t+1}) - \bar{E}^i(R_{k,t+1}) &= 1 + E_t^i(d_{k,t+1}) - \bar{E}^i(d_{k,t+1}) - q_{k,t} + \bar{E}^i(q_{k,t}) \\ E_t^i(R_{p,t+1}^{i,j}) - \bar{E}^i(R_{p,t+1}^{i,j}) &= 1 + E_t^i(d_{p,t+1}^{i,j}) - \bar{E}^i(d_{p,t+1}^{i,j}) - q_{p,t}^{i,j} + \bar{E}^i(q_{p,t}^{i,j}) \end{aligned} \quad (40)$$

where we used the approximation of returns (7) with $d_{k,t+1} = \log(D_{k,t+1}) - \log(D)$, $q_{k,t} = \log(Q_{k,t}) - \log(Q)$ are the log-deviations of dividends and asset prices at the country level from their average, $d_{p,t+1}^{i,j} = \sum_{k=1}^N \tilde{w}_{k,t}^{i,j} d_{k,t+1}$ and $q_{p,t+1}^{i,j} = \sum_{k=1}^N \tilde{w}_{k,t}^{i,j} q_{k,t+1}$ are the fund-specific weighted averages.

Noting that $\frac{a_{k,t}^{i,j} - \bar{E}^i(a_{k,t}^{i,j})}{\bar{E}^i(a_{k,t}^{i,j})}$ can be approximated as $\log(a_{k,t}^{i,j}) - \log(\bar{E}^i(a_{k,t}^{i,j}))$, and that $a_{k,t}^{i,j} = A_{k,t}^{i,j}/\Omega_t^i$, with $A_{k,t}^{i,j}$ the total capital invested by investor i in country k through fund j, we can run the following panel regression:

$$\log(A_{k,t}^{i,j}) = \beta E_t^i(d_{k,t+1}) + \delta E_t^i(d_{p,t+1}^{i,j}) - \delta q_{p,t}^{i,j} + \lambda_{k,t} + \lambda_t^i + \lambda_k^{i,j} + \epsilon_{k,t}^i \quad (41)$$

with

$$\begin{aligned} \lambda_{k,t} &= -\beta q_{k,t} + \beta + \delta \\ \lambda_t^i &= \theta[E_t^i(\mathcal{R}_{p,t+1}^i) - \bar{E}^i(\mathcal{R}_{p,t+1}^i)] + \log(\Omega_t^i) \\ \lambda_k^{i,j} &= -\beta[\bar{E}^i(d_{k,t+1}) - \bar{E}^i(q_{k,t})] - \delta[\bar{E}^i(d_{p,t+1}^{i,j}) - \bar{E}^i(q_{p,t}^{i,j})] + \log(\bar{E}^i(a_{k,t}^{i,j})) \end{aligned} \quad (42)$$

$\lambda_{k,t}$ are country-time fixed effects that capture the impact of country-k asset prices, λ_t^i are investor-time fixed effects that capture the effect of the investor's expectations relative to her whole portfolio and of the investor's wealth, $\lambda_k^{i,j}$ are country-investor-fund fixed effects that capture the impact of investor ex ante expectations on country k and the impact of investor ex ante expectations on j's portfolio. The component of capital flows due to the expectations on country k is $\beta E_t^i(d_{k,t+1})$ and $\delta E_t^i(d_{p,t+1}^{i,j})$ represents the spillovers arising from expectations on the other countries in the portfolio. We cannot account for $q_{p,t}^{i,j}$ through the fixed effects, so we add it as a control. Finally, $\epsilon_{k,t}^{i,j}$ is an error term.

In order to disentangle the portfolio reallocation spillovers from the co-ownership spillovers, we need to identify $\frac{Cov(R_{k,t+1},R^{i,j}_{p,k^-,t+1})-Cov(R_{k,t+1},\mathcal{R}^i_{p,j^-,t+1})}{V(R^{i,j}_{p,t+1})}\phi$, the part of δ that is due to the portfolio reallocation. Using a measure of $\frac{Cov(R_{k,t+1},R^{i,j}_{p,k^-,t+1})}{V(R^{i,j}_{p,t+1})}$ and $\frac{Cov(R_{k,t+1},\mathcal{R}^i_{p,j^-,t+1})}{V(R^{i,j}_{p,t+1})}$, we can estimate the following modified version that includes an interaction term:

$$\log(A_{k,t}^{i,j}) = \beta E_t^i(d_{k,t+1}) + \eta E_t^i(d_{p,t+1}^{i,j}) - \phi \frac{Cov(R_{k,t+1}, R^{i,j}_{p,k^-,t+1}) - Cov(R_{k,t+1}, \mathcal{R}^i_{p,j^-,t+1}}{V(R^{i,j}_{p,t+1})} E_t^i(d_{p,t+1}^{i,j})$$
$$- \delta q_{p,t}^{i,j} + \lambda_{k,t} + \lambda_t^i + \lambda_k^{i,j} + \epsilon_{k,t}^i \qquad (43)$$

The spillovers arising from co-ownership are given by η while the spillovers arising from portfolio reallocation are given by ϕ.

5.2 Reallocation and Co-ownership spillovers

In what follows, we implement the estimation of Equation (43) and provide estimates for the parameters β, δ, η and ϕ that are necessary to identify portfolio reallocation and co-ownership spillovers.

To do so, we estimate a slightly modified version of Equation (43)

$$\ln\left(A_{k,t}^{i,j}\right) = \beta E_t^i g_k^{\text{next year}} + \delta E_t^i g_p^{j,\text{next year}} + \gamma_1 \Delta \log(Q_{p,t}^j) + \gamma_2 \Delta \log(A_t^j)$$
$$+ \lambda_{k,t} + \lambda_t^i + \lambda_k^{i,j} + \epsilon_{k,t}^{i,j}, \qquad (44)$$

where $E_t^i g_k^{\text{next year}}$ and $E_t^i g_p^{j,\text{next year}}$ proxy for the expected deviation of dividends from their mean at the country and fund level, and the log-change in the fund-relevant equity price $\Delta Q_{p,t}^j$ proxies for the log-deviation of equity prices from their average. The coefficient β corresponds to the the effect on capital flows stemming from country k expectations and the coefficient δ measures the effect of spillovers arising from expectations on other countries in the portfolio. The coefficient γ_1 is equal to δ in theory but in practice it also captures potential valuation effects and portfolio rebalancing by the fund. β and δ are therefore identified through the impact of expectations. The aggregate capital flows $\Delta \log(A_t^j)$ is added to improve the identification of δ, as in Table 1. We add more controls in the form of lags of the control variables $\Delta \log(Q_{p,t}^j)$ and $\Delta \log(A_t^j)$

Table 3, column (1) reports the results of regression (44). The effect of spillovers from expectations δ is positive and significant. Quantitatively, a 1 percentage point increase in the aggregate investor expectations increases the capital allocation to country k by fund j by about 19 percent. The effect on capital flows stemming from the country expectations β is about 4 times lower: a 1 percentage point increase in the GDP growth expectation for country k increases capital allocation to country k by about 5.4 percent.

In order to disentangle the portfolio reallocation spillovers from the co-ownership spillovers, we examine the empirical counterpart to Equation (43). We proxy for the scaled conditional covariance of the country return k with the fund-level return excluding country k and for the scaled conditional covariance of the country return k with the investor-level return excluding fund j by using the surprises in GDP growth at the investor level.

Define the aggregate fund-level growth, the aggregate fund-level growth excluding country k and the aggregate investor-level growth excluding fund j respectively as follows:

$$\begin{aligned} g_{p,k-,t}^{j,\text{next year}} &= \sum_{l\neq k, l\in K(i,j)} \frac{w_{l,t}^{i,j}}{\sum_{l\neq k, l\in K(i,j)} w_{l,t}^{i,j}} g_{l,t}^{\text{next year}}, \\ g_{p,t}^{j,\text{next year}} &= \sum_{l\in K(i,j)} \frac{w_{l,t}^{i,j}}{\sum_{l\in K(i,j)} w_{l,t}^{i,j}} g_{l,t}^{\text{next year}}, \\ g_{p,j-,t}^{i,\text{next year}} &= \sum_{l\neq j, l\in J(i)} \frac{A_t^{i,l}}{\sum_{l\neq j, l\in J(i)} A_t^{i,l}} g_{p,t}^{j,\text{next year}}, \end{aligned} \tag{45}$$

where $w_{l,t}^{i,j}$ is mutual fund j's allocation to country l, $A_t^{i,l}$ is fund l total assets under management and $g_{l,t}^{\text{next year}}$ is the country l's next year GDP growth in percent. We measure this realized growth rate as the first release in the IMF World Economic Outlook published in April of the following year, as Benhima and Bolliger (2023). We then define the investor surprises as $FE_{p,k-,t}^{i,j} = g_{p,k-,t}^{j,\text{next year}} - E_t^i(g_{p,k-,t}^{j,\text{next year}})$, the error on the partial fund-level growth, $FE_{p,t}^{i,j} = g_{p,t}^{j,\text{next year}} - E_t^i(g_{p,t}^{j,\text{next year}})$, the error on the full fund-level growth, $FE_{p,j-,t}^{i} = g_{p,j-,t}^{i,\text{next year}} - E_t^i(g_{p,j-,t}^{i,\text{next year}})$ the error on the partial investor-level growth and $FE_{k,t}^{i} = g_{k,t}^{\text{next year}} - E_t^i(g_{k,t}^{\text{next year}})$, the error on country growth. We then compute the scaled conditional covariances by country and fund-investor pair, $\frac{Cov^i(FE_k^i, FE_{p,k-}^{i,j})}{Var(FE_p^{i,j})}$ and $\frac{Cov^i(FE_k^i, FE_{p,j-}^{i})}{Var(FE_p^{i,j})}$. We interact this measure with the aggregate expectation of GDP growth in country k in the following regression

$$\begin{aligned}
\ln\left(A^{ij}_{k,t}\right) &= \beta E^i_t g^{\text{next year}}_k + \eta E^i_t g^{j,\text{next year}}_{p,t} \\
&+ \left(-\phi_1 \frac{Cov^i(FE^i_k, FE^{i,j}_{p,k-})}{Var(FE^{i,j}_p)} + \phi_2 \frac{Cov^i(FE^i_k, FE^i_{p,j-})}{Var(FE^{i,j}_p)}\right) \times E^i_t g^{j,\text{next year}}_{p,t} \\
&+ \gamma_1 \Delta \log(Q^j_{p,t}) + \gamma_2 \Delta \log(A^j_t) + \lambda_{k,t} + \lambda^i_t + \lambda^{i,j}_k + \epsilon^{i,j}_{k,t}.
\end{aligned} \tag{46}$$

This specification allows us to distinguish the portfolio reallocation spillover parameters ϕ_1 and ϕ_2 from the co-ownership spillovers parameter η. While in theory, $\phi_1 = \phi_2 = \phi$, we allow these parameters to differ.

The results of regression (46) are shown in Table 3, Column (2).The coefficient η, which reflects the effect on capital flows arising from co-ownership spillovers, is positive and significant. The coeffcient β also remains stable and significant. However, the coefficients on the interaction terms $-\phi_1$ and ϕ_2, which reflect the portfolio reallocation spillovers, are not significant. This suggests that, in practice, the portfolio reallocation motive is not very strong. The η coefficient happens to be very close to δ at about 0.25. This means that on average portfolio spillovers are mostly co-ownership spillovers.

In column (3), we decompose the portfolio expectations into its average and the granular term, in order to address the missing variable bias discussed above. The coefficient of the granular term, which is an unbiased estimate of η, is approximately equal to 0.16, which is lower than the biased coefficient, but remains large significant and is still three times larger than β. This lends further support to the prediction that co-ownership spillovers adversely affect capital flows because certain countries compose a large share of portfolios.

Using our estimates of β and η from the last columns, we can get an estimate of the portfolio friction parameter p. To do so, we apply Corollary 4.1's prediction that $\beta/\eta = p/(1-p)$. This yields $p = \beta/(\eta + \beta) = 0.057/(0.164 + 0.057) = 0.26$. This means that mutual funds update their portfolios every 4 months on average. As a comparison, Bacchetta and van Wincoop (2017) estimate that $p = 0.04$ using a model with a Calvo-type portfolio friction. This implies an average portfolio updating span of two years. However, their estimate is based on aggregate portfolio and equity price data for the US. Our estimate concerns individual mutual funds. In the presence of multiple intermediates, our parameter underestimates the aggregate portfolio stickiness. If we assume for instance that investors update their portfolios with the same frequency as mutual funds, then the aggregate updating frequency would be $p^2 = 0.066$, which is closer to their estimate.

	(1)	(2)	(3)
VARIABLES	$\log(A^{i,j}_{k,t})$	$\log(A^{i,j}_{k,t})$	$\log(A^{i,j}_{k,t})$
$E^i_t(g^{\text{next year}}_k)$	0.054*	0.058*	0.057*
	(0.030)	(0.030)	(0.030)
$E^i_t(g^{j,\text{next year}}_p)$	0.193***	0.256***	
	(0.046)	(0.059)	
$\bar{E}^i_t(g^{j,\text{next year}}_p)$			0.283***
			(0.062)
$\Gamma^{i,j}_t$			0.164***
			(0.053)
$\frac{Cov(g_k, g^j_{p,k-})}{Var(g^j_p)} \times E^i_t g^{j,\text{next year}}_p$		-0.016	-0.027
		(0.045)	(0.044)
$\frac{Cov(g_k, g^i_{p,j-})}{Var(g^j_p)} \times E^i_t g^{j,\text{next year}}_p$		-0.036	-0.031
		(0.022)	(0.022)
$\Delta \log(A^j_t)$	-0.048***	-0.053***	-0.064***
	(0.018)	(0.018)	(0.019)
$\Delta \log(A^j_{t-1})$	-0.030**	-0.030**	-0.037**
	(0.014)	(0.015)	(0.015)
$\Delta \log(Q^{i,j}_t)$	-1.086*	-1.110*	-1.085*
	(0.579)	(0.596)	(0.597)
$\Delta \log(Q^{i,j}_{t-1})$	-0.406	-0.397	-0.385
	(0.620)	(0.629)	(0.630)
Observations	26,513	26,192	26,192
R-squared	0.917	0.918	0.919
Country-Fund FE	Yes	Yes	Yes
Country-time FE	Yes	Yes	Yes
Investor-time FE	Yes	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3: Spillovers due to Portfolio Reallocation and Co-ownership

Note: The dependent variable is the log total capital invested by investor i in country k through fund j on month t. Column (1) reports results for regression Equation (44). Column (2) reports the regression results of Equation (46). Column (3) reports regression results with a decomposition of the simple average and granular component of investor expectations.

	(1)	(2)	(3)
	$\log(A^{i,j}_{k,t})$	$\log(A^{i,j}_{k,t})$	$\log(A^{i,j}_{k,t})$
VARIABLES	Global	GEM	Regional
$E^i_t(g^{\text{next year}}_k)$	0.057	0.178***	0.004
	(0.100)	(0.042)	(0.033)
$\bar{E}^i_t(g^{j,\text{next year}}_p)$	0.555***	0.890**	0.103**
	(0.133)	(0.358)	(0.047)
$\Gamma^{i,j}_t$	0.708***	0.730*	0.148**
	(0.195)	(0.436)	(0.069)
$\frac{Cov(g_k,g^j_{p,k-})}{Var(g^j_p)} \times E^i_t g^{j,\text{next year}}_p$	0.057	-0.066	0.102
	(0.098)	(0.043)	(0.064)
$\frac{Cov(g_k,g^i_{p,j-})}{Var(g^j_p)} \times E^i_t g^{j,\text{next year}}_p$	-0.171***	0.004	-0.001
	(0.059)	(0.023)	(0.032)
Observations	6,727	5,890	11,796
R-squared	0.956	0.945	0.930
Country-Fund FE	Yes	Yes	Yes
Country-time FE	Yes	Yes	Yes
Investor-time FE	Yes	Yes	Yes
Controls	Yes	Yes	Yes

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 4: Spillovers due to Portfolio Reallocation and Co-ownership by fund type

Note: The dependent variable is the log total capital invested by investor i in country k through fund j on month t. The specification is the one used in columns (3) of Table 3 for different fund groups (the coefficients of the control variables are not shown for parsimony). Standard errors are clustered at the country-year and investor-year level.

Table 4 applies the specification of column (3) to identify the parameters for different fund types, based on their declared scope. The co-ownership spillovers are sizeable for all funds, but are particularly strong for Global and Global Emerging Market (GEM) funds: these funds have a large estimated η of about 0.7, while this coefficient is lower in Regional funds at about 0.15. Global and Regional funds react very little to country expectations with an allocation updating probability that is not distinguishable from zero as β is insignificant (columns (1) and (3)). Global Emerging Market funds are more active, with a probability $p = 0.178/(0.730+0.178) = 0.20$ (column (3)), which is consistent with our previous findings. Note that the probability p appears smaller when we allow for heterogeneous coefficients across funds: the most active funds have a probability of 0.20 while the average probability, when assuming homogeneous coefficients, is 0.26.

5.3 Spillovers from investor-wide expectations

Finally, we focus on the identification of θ, the elasticity of capital flows to the investor-wide expectations.

To identify θ, we expand Equation (43). To do so, we first approximate the investor-wide surprises in returns as follows

$$E_t^i(\mathcal{R}_{p,t+1}^i) - \bar{E}^i(\mathcal{R}_{p,t+1}^i) = 1 + E_t^i(d_{p,t+1}^i) - \bar{E}^i(d_{p,t+1}^i) - q_{p,t}^i + \bar{E}^i(q_{p,t}^{i,j}) \tag{47}$$

where $d_{p,t+1}^i = \sum_{j=1}^{J(i)} \frac{A_t^{i,j}}{\sum_{j=1}^{J(i)} A_t^{i,j}} d_{p,t+1}^j$ and $q_{p,t+1}^i = \sum_{j=1}^{J(i)} \frac{A_t^{i,j}}{\sum_{j=1}^{J(i)} A_t^{i,j}} q_{p,t+1}^j$ are the investor-specific weighted averages of dividends and prices. Substituting into λ_t^i, and then into (43), we obtain

$$\begin{aligned} \log(A_{k,t}^{i,j}) = {} & \beta E_t^i(d_{k,t+1}) + \eta E_t^i(d_{p,t+1}^{i,j}) + \theta E_t^i(d_{p,t+1}^i) \\ & - \phi \frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j}) - Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^i}{V(R_{p,t+1}^{i,j})} E_t^i(d_{p,t+1}^{i,j}) \\ & - \delta q_{p,t}^{i,j} - \theta q_{p,t}^i + \log(\Omega_t^i) + \lambda_{k,t} + \lambda_k^{i,j} + \epsilon_{k,t}^i \end{aligned} \tag{48}$$

with now $\lambda_k^{i,j}$ standing for

$$\begin{aligned} \lambda_k^{i,j} = {} & -\beta[\bar{E}^i(d_{k,t+1}) - \bar{E}^i(q_{k,t})] - \delta[\bar{E}^i(d_{p,t+1}^{i,j}) - \bar{E}^i(q_{p,t}^{i,j})] + \log(\bar{E}^i(a_{k,t}^{i,j})) \\ & - \bar{E}^i(d_{p,t+1}^i) + \bar{E}^i(q_{p,t}^{i,j}) \end{aligned} \tag{49}$$

This equation is brought to the data in the following form:

$$\ln\left(A_{k,t}^{ij}\right) = \beta E_t^i g_k^{\text{next year}} + \eta E_t^i g_{p,t}^{j,\text{next year}} + \theta E_t^i \mathsf{g}_{p,t}^{i,\text{next year}}$$
$$+ \left(-\phi_1 \frac{Cov^i(FE_k^i, FE_{p,k-}^{i,j})}{Var(FE_p^{i,j})} + \phi_2 \frac{Cov^i(FE_k^i, FE_{p,j-}^i)}{Var(FE_p^{i,j})}\right) \times E_t^i g_{p,t}^{j,\text{next year}}$$
$$+ \gamma_1 \Delta \log(Q_{p,t}^j) + \gamma_2 \Delta \log(\mathtt{Q}_{p,t}^i) + \gamma_3 \Delta \log(A_t^j) + \gamma_4 \Delta \log(\mathtt{A}_t^i) + \lambda_{k,t} + \lambda_k^{i,j} + \epsilon_{k,t}^{i,j}. \tag{50}$$

The difference between this equation and Equation (43) is that it includes a measure of investor GDP growth expectation at the investor level $E_t^i \mathsf{g}_{p,t}^{i,\text{next year}}$, and a measure of equity price change at the investor level $\Delta \log(\mathtt{Q}_{p,t}^i)$. This aggregate GDP growth expectation is computed as

$$E_t^i g_{p,t}^{i,\text{next year}} = \sum_{j \in J(i)} \frac{A_{t-1}^{i,j}}{\sum_{j \in J(i)} A_{t-1}^{i,j}} E_t^i g_{p,t}^{i,\text{next year}}, \tag{51}$$

where $E_t^i g_{p,t}^{i,\text{next year}}$ is defined in (51) and $A_{t-1}^{i,j}$ is the past value of total assets under management by fund j. The aggregate price chnages is computed in the same way using the log-changes in the MSCI indices at the fund level and aggregating using the funds' past assets under management. We do not have a good measure of the investor's total wealth Ω_t^i, so we omit this variable.

Another difference is that we do not include the investor-time fixed effects, as they would absorb the investor-time specific expectation $E_t^i \mathsf{g}_{p,t}^{i,\text{next year}}$ that we need to identify θ. In the absence of these fixed effects, we cannot account for Ω_t^i, the investor total wealth, for which we do not have a good measure. This means that we cannot account for funding shocks, which are an important driver of capital flows. The global drivers of these funding shocks are accounted for by the country-time fixed effects, but those do not account for the investor's own wealth dynamics. The latter are problematic for identification if they correlate with expectations. The main potential drivers of this correlation are equity prices, as they are both signals that could be used to form expectations and a driver of investor wealth. However, we already control for $\Delta \log(\mathtt{Q}_{p,t}^i)$, which also account for valuation effects and portfolio rebalancing by investors. In addition, we also control for the total capital flows $\Delta \log(\mathtt{A}_t^i)$ that are invested into the set of countries where the investor is active. This accounts for a potential reverse causality from capital flows to growth and growth expectations. This variable is computed using the total flows at the country level, which we then aggregate using the fund's past cross-country allocations, and taking the log-change.

	(1)	(2)	(3)	(4)
	$\log(A^{i,j}_{k,t})$	$\log(A^{i,j}_{k,t})$	$\log(A^{i,j}_{k,t})$	$\log(A^{i,j}_{k,t})$
VARIABLES	All	Global	GEM	Regional
$E^i_t(g^{\text{next year}}_k)$	0.001	-0.034	0.038	0.021
	(0.028)	(0.120)	(0.063)	(0.035)
$\bar{E}^i_t(g^{j,\text{next year}}_p)$	0.300***	0.417***	0.527***	0.104**
	(0.059)	(0.079)	(0.106)	(0.045)
$\Gamma^{i,j}_t$	0.207***	0.346**	0.621***	0.106*
	(0.057)	(0.147)	(0.228)	(0.060)
$\frac{Cov(g_k,g^j_{p,k-})}{Var(g^j_p)} \times E^i_t g^{j,\text{next year}}_p$	-0.012	0.007	-0.006	0.061
	(0.050)	(0.081)	(0.075)	(0.057)
$\frac{Cov(g_k,g^i_{p,j-})}{Var(g^j_p)} \times E^i_t g^{j,\text{next year}}_p$	-0.043*	-0.128**	-0.013	0.004
	(0.025)	(0.055)	(0.028)	(0.025)
$\bar{E}^i_t(\mathsf{g}^{i,\text{next year}}_p)$	0.181	-0.264	-0.034	0.143
	(0.136)	(0.321)	(0.290)	(0.129)
Γ^i_t	0.413*	-0.320	-0.094	0.060
	(0.221)	(0.419)	(0.745)	(0.195)
Observations	25,335	6,101	5,833	11,785
R-squared	0.909	0.942	0.918	0.923
Country-Fund FE	Yes	Yes	Yes	Yes
Investor-time FE	Yes	Yes	Yes	Yes
Country-time FE	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 5: Investor-wide spillovers

Note: The dependent variable is the log total capital invested by investor i in country k through fund j on month t. The specification is the one used in columns (3) of Table 3 for different fund groups (the coefficients of the control variables are not shown for parsimony). Standard errors are clustered at the country-year and investor-year level.

As before, we also include the first lag of the control variables. Finally, for the same reasons discussed earlier, we decompose $E_t^i g_{p,t}^{j,\text{next year}}$, but also $E_t^i \mathrm{g}_{p,t}^{i,\text{next year}}$ into their granular and common components. The variables $\Gamma_t^{i,j}$ and $\bar{E}_t^i g_{p,t}^{j,\text{next year}}$ are defined in Equation (54) and (4) and Γ_t^i and $\bar{E}_t^i g_{p,t}^{i,\text{next year}}$ are defined in a similar way:

$$\Gamma_t^i = E_t^i g_{p,t}^{i,\text{next year}} - \bar{E}_t^i g_{p,t}^{j,\text{next year}}. \tag{52}$$

with

$$\bar{E}_t^i g_{p,t}^{i,\text{next year}} = \left(\sum_{j \in J(i)} \sum_{k \in K(i,j)} \frac{A_{t-1}^{i,j}}{\sum_{j \in J(i)} A_{t-1}^{i,j}} w_{k,t-1}^{i,j} \right) \frac{1}{\mathtt{N}^i} \sum_{k \in K(i,j)} \sum_{j \in J(i)} E_t^i g_{k,t}^{\text{next year}}, \tag{53}$$

where $\mathtt{N}^i$ is the number of elements in $\bigcup_{j \in J(i)} K(i,j)$.

We perform this regression on the whole sample and on sub-samples based on the fund type. We consider the coefficient of Γ_t^i to be the best possible estimate of θ. The results are reported in Table 5. In column (1), which gives the result for the whole sample, θ is estimated to be equal to 0.43 with a 10% significance level. However, this coefficient is not significant in the subsamples (columns (2) to (4)). This could be explained by an endogeneity bias driven by a heterogeneous response of fund types to global shocks. Suppose that a monetary contraction in the US drives capital flows away from GEM funds into Global funds. This generates a drop in growth expectations for GEM funds relative to Global funds. This generates a positive correlation between investor-wide expectations and capital flows if GEM funds are concentrated among some investors. By running a regression by fund type, we avoid this issue. To check whether this hypothesis is valid, we introduce fund type-time fixed effect in the full sample regression in column (5). The coefficient of Γ_t^i becomes insignificant. Therefore, we consider the results by fundtype to be reliable and conclude that θ is indistinguishable from zero.

6 Quantifying Co-ownership Spillovers

Equation (34) provides a decomposition of capital flows to country k (as a percentage of total managed wealth) into the contribution of country-specific expectations, the contribution of global expectations and the contribution of the granular terms, which include the co-ownership spillovers. Because capital flows have many drivers besides expectations on GDP growth, we focus on the contribution of co-ownership spillovers to the variance of capital flows stemming from GDP growth expectations, which we call the expectation-driven capital flows.

We have shown that, in our model, the co-ownership spillovers are inefficient as they arise only in the presence of portfolio stickiness ($p < 1$). The data has shown us that portfolio stickiness is pervasive, confirming a hypothesis that has been previously made in the theoretical literature. It is therefore highly relevant to evaluate the contribution of this friction to expectation-driven capital flow volatility. Importantly, as Equation (34) and Corollary 4.1 have shown, co-ownership spillovers impact capital flows through the coefficient η, which we have estimated in the previous section, and through the granular terms.

Define $\Gamma^a_{k,t}$ as the capital flows due to the co-ownership spillovers. We have assumed, in our baseline empirical analysis, that the coefficients $\eta^{i,j}_k$ are homogenous across countries and funds. We make the same assumption here, so that $\eta^{i,j}_k = \eta$. This yields $\Gamma^a_{k,t} = \eta\Gamma_{k,t}$ where $\Gamma_{k,t}$ is a measure of the granular expectations at the country level:

$$\Gamma_{k,t} = \sum_{i=1}^{M}\sum_{j=1}^{J(i)} \sigma^{i,j}_{k,t}\Gamma^{i,j}_t \tag{54}$$

We can then estimate $\Gamma^a_{k,t}$ using the data. We have already a proxy for $\Gamma^{i,j}_t$ from our empirical analysis. The parameter η has been identified in Section 5 to be equal to 0.164 on average. Finally, $\sigma^{i,j}_{t,k}$ can be estimated as the average share of fund j in the total investment in country k: $\sum_{t=1}^{T} A^{i,j}_{k,t}/\sum_{t=1}^{T} A_{k,t}$. This share is fixed so that the variation in our co-ownership spillovers come entirely from variations in the granular residual of expectations. We can then identify the contribution of co-ownership spillovers to the aggregate capital flows.

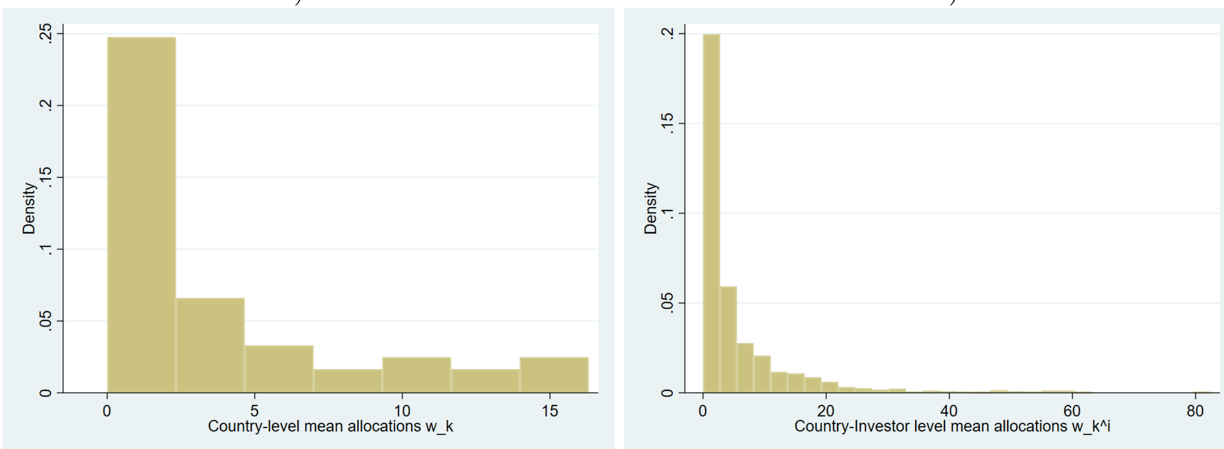
Using the definition of $\Gamma^{i,j}_t$, we can write:

$$\begin{aligned}
\Gamma_{k,t} &= \sum_{k'=1}^{N}\sum_{i=1}^{M} l^i_{k',t}\left(\sum_{j=1}^{J(i)} \sigma^{i,j}_{k,t}\tilde{w}^{i,j}_{k',t}\right) \\
&= \sum_{k'=1}^{N}\sum_{i=1}^{M} \tilde{w}^i_{k,k',t} l^i_{k',t} \\
&= \left[\sum_{k'=1}^{N} \tilde{w}_{k,k',t} l_{k',t} + \sum_{k'=1}^{N}\sum_{i=1}^{M} \tilde{w}^i_{k,k',t}(l^i_{k',t} - l_{k',t})\right]
\end{aligned} \tag{55}$$

Here we distinguish between the the average country-specific component of expectations across investors for country k', $l_{k',t} = (\sum_{i=1}^{M} l^i_{k',t})/M$ and its investor-specific component

Figure 1: Distribution of country allocations

a) b)



$l^i_{k',t} - l_{k',t}$. These expectations are respectively weighted by the shares

$$\tilde{w}_{k,k',t} = \left(\sum_{i=1}^{M} \sum_{j=1}^{J(i)} \sigma^{i,j}_{k,t} \tilde{w}^{i,j}_{k',t} \right)$$
$$\tilde{w}^i_{k,k',t} = \left(\sum_{j=1}^{J(i)} \sigma^{i,j}_{k,t} \tilde{w}^{i,j}_{k',t} \right) \tag{56}$$

$\tilde{w}_{k,k',t}$ is a weighted average of country k''s allocations across all funds, where the weights are the represented by the importance of a given fund in the total flows to country k. The aggregate expectation shocks on country k' $l_{k',t}$ will matter to country k if the funds that channel a large share of country k investment also invest a lot in country k'. $\tilde{w}^i_{k,k',t}$, on the other hand, is a weighted average of country k''s allocations across investor i's funds, where the weights are the represented by the importance of a given fund in the total flows to country k. Investor i's idiosyncratic expectation shocks on country k' $l^i_{k',t} - l_{k',t}$ will matter to country k if the funds managed by i that channel a large share of country k investment also invest a lot in country k'.

As shown by Gabaix (2011), the aggregate relevance of expectations depends on the nature of the distribution of the shares $\tilde{w}_{k,k',t}$ and $\tilde{w}^i_{k,k',t}$. If the shares $\tilde{w}_{k,k',t}$ are fat-tailed, that is, if some countries have disproportionate weight in global portfolios, then their country-specific expectation component $l_{k,t}$ will matter. Similarly, the idiosyncratic expectations will matter in the aggregate if the investor-specific country shares $\tilde{w}^i_{k,k',t}$' distribution is fat-tailed. Figure 6 represents the distribution of the country allocations at the global level $\tilde{w}_k$

(panel a)), computed as the weighted average of country allocation across all funds, and the distribution of the country allocations at the investor level $\tilde{w}_k^i$ (panel b)), computed as the weighted average of country allocation across each investor's funds. These distributions show that a few shares are very large.

We compute an estimate of $\Gamma_{k,t}^a = \eta\Gamma_{k,t}$ based on Equation (**??**) using $\eta = 0.164$, consistently with the baseline analysis, and using our expectation and capital flow data to compute $\Gamma_{k,t}$. To measure expectations, we use the growth expectations $E_t^i g_k^{\text{next year}}$. However, since we have many missing expectations, we expand the expectation data as much as possible by imputing expectations when we do not observe them. To do so, we fit an ad hoc expectation process to our data and impute fictitious expectation data when that data is missing. See the Appendix for details.

To isolate the role of expectations from that of the country weights, we examine the terms $\Delta\Gamma_{k,t}^a = \eta\Delta\Gamma_{k,t}$, with

$$\Delta\Gamma_{k,t} = \sum_{i=1}^{M} \sum_{k' \in \kappa(i)} \tilde{w}_{k,k',t-1}^i (l_{k',t}^i - l_{k',t-1}^i) \tag{57}$$

where $\kappa(i)$ is the set of countries for which we observe investor i's expectations or impute expectations. This is the innovation in capital flows to country k that is due to co-ownership spillovers. Indeed, the weights $\tilde{w}_{k,k',t-1}^i$ are kept equal to their past value. Because there are some countries in which investor i invests and for which we do not observe expectations, the magnitude of this term is under-estimated. Our estimates of the variance of $\Delta\Gamma_{k,t}^a$ will thus be conservative.

To compare these co-ownership spillovers to the total expectation-driven flows to country k, we compute also the common terms $\Delta W_{k,t}^a = (\beta + \eta)\Delta W_{k,t}$ and the idiosyncratic terms $\Delta l_{k,t}^a = \beta\Delta l_{k,t}$, where $\Delta W_{k,t}$ and $\Delta l_{k,t}$ are computed as

$$\begin{aligned} \Delta W_{k,t} &= \sum_{i=1}^{M} \left(\sum_{j=1}^{J(i)} \sigma_{k,t}^{i,j} \right) (W_t^i - W_{t-1}^i) \\ \Delta l_{k,t} &= \sum_{i=1}^{M} \left(\sum_{j=1}^{J(i)} \sigma_{k,t}^{i,j} \right) (l_{k,t}^i - l_{k,t-1}^i) \end{aligned} \tag{58}$$

and where $\beta = 0.057$ as in our baseline estimation.

Expectations			
Variance	$V\left(\Delta\Gamma_t^k\right)$	$V\left(\Delta W_{k,t}\right))$	$V\left(\Delta l_{k,t}\right))$
Value	.012	.022	.111
	[.005,0.023]	[.006,.044]	[0,.637]
Contribution	6%	15%	75%
	[2%,42%]	[3%,61%]	[0%,95%]

Implied capital flows			
Homegeneous case			
Coefficients	η	$\eta+\beta$	β
	.164	.218	.057
Variance	$V\left(\Delta\Gamma_{k,t}^a\right)$	$V\left(\Delta W_{k,t}^a\right))$	$V\left(\Delta l_{k,t}^a\right))$
Value	.0004	.0013	.0003
	[.0002,.0007]	[.0010,.0023]	[.0001,.0009]
Contribution	19%	64%	14%
	[11%,27%]	[50%,73%]	[4%,33%]
Heterogeneous case			
Coefficients	η	$\eta+\beta$	β
Global	.71	.71	0
GEM	.73	.91	.18
Regional	.15	.15	0
Variance	$V\left(\Delta\Gamma_{k,t}^a\right)$	$V\left(\Delta W_{k,t}^a\right))$	$V\left(\Delta l_{k,t}^a\right))$
Value	.0037	.0085	.0001
	[.0011,.0069]	[.0010,0.0199]	[0,.0045]
Contribution	31%	63%	3%
	[12%,49%]	[44%,79%]	[0%,19%]
Variance	$V\left(\Delta\Gamma_{k,t}^{a'}\right)$	$V\left(\Delta W_{k,t}^a\right))$	$V\left(\Delta l_{k,t}^{a'}\right))$
Value	.0037	.0085	.0012
	[.0010,.0066]	[.0010,0.0199]	[0,.0114]
Contribution	28%	61%	9%
	[10%,49%]	[44%,75%]	[0%,35%]

Table 6: Variance decomposition of expectations and expectation-driven capital flows

Note: We report the median variances of expectations and implied capital flows across countries, as well as the 10^{th} and 90^{th} percentile (in brackets). The contributions are the ratio of the variance to the sum of the variances of the granular, common and idiosyncratic terms.

We assume that $\theta = 0$, consistently with our empirical analysis. Then, according to Equation (34), the innovation in the expectation-driven capital flows, which we denote $\tilde{a}_{k,t}$, can be decomposed into the granular, common and idiosyncratic components as follows:

$$\frac{\tilde{a}_{k,t} - \tilde{a}_{k,t-1}}{\bar{E}(\tilde{a}_{k,t})} = \Delta\Gamma^a_{k,t} + \Delta W^a_{k,t} + \Delta l^a_{k,t} \tag{59}$$

The contribution of each of these terms to the variance of expectation-driven flows is given in Table 6.

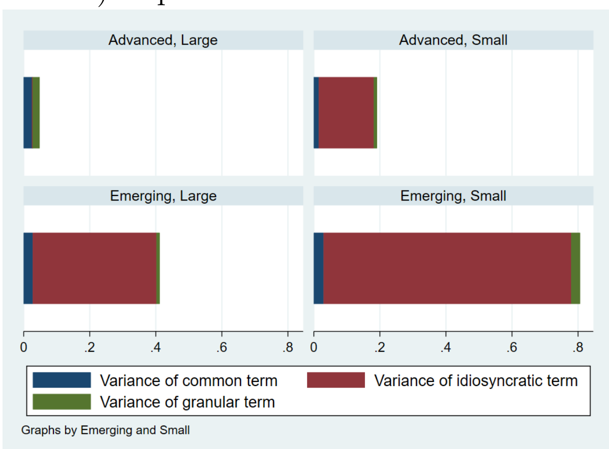
First consider the upper part of Table 6, which focuses on the decomposition of expectations. The largest component of expectations is the idiosyncratic term with a median contribution of 75%, although it is highly heterogeneous. Panel a) of Figure 2 shows the variance decomposition of expectations for Emerging, Advanced, Small and Large countries. We define a country as “Large” when its average share in portfolios is higher than 7.5%. The Large countries include the United States, the United Kingdom, Japan, Germany, France, Switzerland, the Russian Federation, South Korea, China and Brazil. The idiosyncratic term is particularly large for Emerging economies, and even more so for Small Emerging economies, as panel a). Large Advanced economies have a negligible contribution of idiosyncratic expectations, probably because their business cycle contribute to the global fluctuations. For Small Advanced economies, the idiosyncratic component is smaller, but its is still dominant. The median contributions of the granular and common components are respectively 6% and 15%. The granular component is not trivial as its variance is equal to 40% of that of the common component. This means that one third of the comovement in expectations is due to the granular term.

Now consider the lower part of Table 6 that describes the contributions of the different components to the expectation-driven capital flows, under the assumption that the β and η coefficients are homogeneous, and under the assumption that they are heterogeneous across funds. In the heterogeneous case, the variance of the granular term is much larger (almost 10 times), because the η coefficients for Global and GEM funds are significantly higher. This is true for both Emerging and Advanced economies, and for both Small and Large countries, which we can see by comparing panels b) and c) of Figure 2.

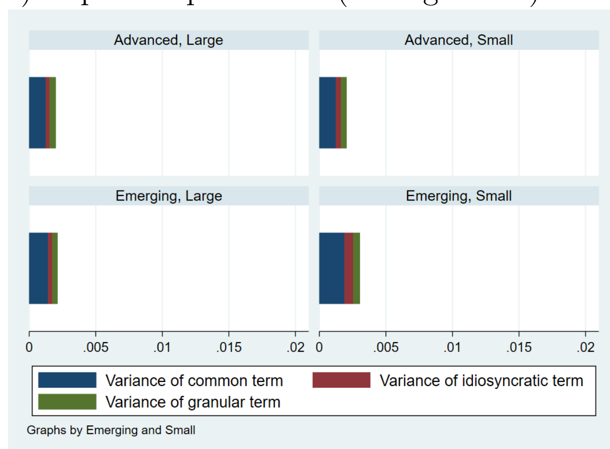
Now consider the contribution of the granular relative to the other terms. In the homogeneous case, because β is low relative to η, now the common and granular terms have a much higher contribution relative to the idiosyncratic term: the granular term (co-ownership spillovers) explain 19% of the variance, and the common term explains 64%. This means that co-ownership spillovers explain about one fourth of the comovement in expectation-driven flows. A similar result holds for the heterogeneous case, with respectively 31% and

Figure 2: Variance decomposition of expectations and expectation-driven capital flows

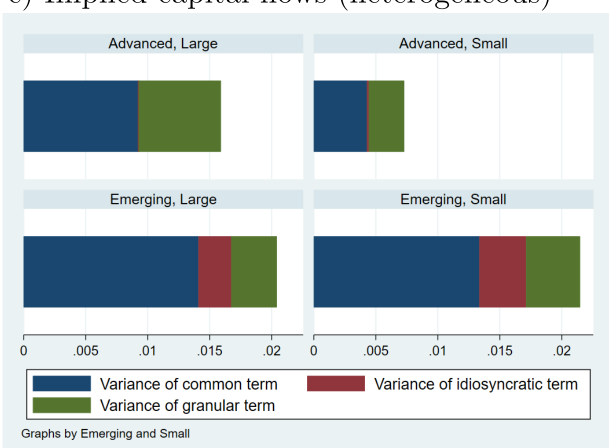
a) Expectations



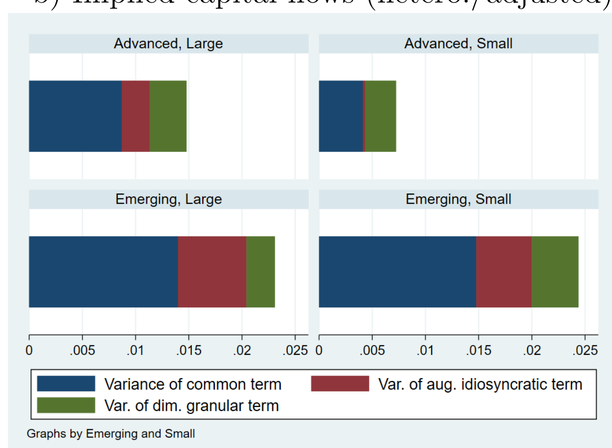
b) Implied capital flows (homogeneous)



c) Implied capital flows (heterogeneous)



b) Implied capital flows (hetero./adjusted)



Note: We report the average variances of expectations and implied capital flows across countries. Panel a) represents the variance of expectations due to $W_{k,t}$, $l_{k,t}$ and $\Gamma_{k,t}$. Panel b) represents the variance of implied capital flows due to $W^a_{k,t}$, $l^a_{k,t}$ and $\Gamma^a_{k,t}$ under the assumption of homogeneous parameters. Panel c) represents the variance of implied capital flows due to $W^a_{k,t}$, $l^a_{k,t}$ and $\Gamma^a_{k,t}$ under the assumption of heterogeneous parameters. Panel d) represents the variance of implied capital flows due to $W^a_{k,t}$, $l^{a'}_{k,t}$ and $\Gamma^{a'}_{k,t}$ under the assumption of heterogeneous parameters.

63%. Here, co-ownership spillovers explain one third of the comovement. Interestingly, the co-ownership spillovers are the largest for Large Advanced economies, followed by Small Emerging, Large Emerging and Small Advanced. The idiosyncratic component is relevant only for Emerging countries, as GEM funds, which are the most active, operate in those countries.

Note however that the large countries' granular term may not necessarily only reflect spillovers from other countries, since the granular term is precisely driven by the expectations about large countries. For instance, the investments of a fund in China could still reflect the expectations about China's growth even though the fund is inactive, just because the expectations about China have a non-trivial impact on the aggregate expectations that drive capital flows to fund. We thus subtract from the granular term the following term:

$$\Gamma_{k,k,t} = \sum_{i=1}^{M} \tilde{w}^i_{k,k,t} l^i_{k,t} \tag{60}$$

This term reflects the impact of the investors' expectation on country k through the granular term. This term should actually be associated to the idiosyncratic term, not to the granular term. We thus compute a diminished granular term: $\Delta\Gamma^{a'}_{k,t} = \eta\Delta(\Gamma_{k,t} - \Gamma_{k,k,t})$, and an augmented idiosyncratic term: $\Delta lk, t^{a'} = \beta\Delta lk, t + \eta\Delta\Gamma_{k,k,t}$. The median contribution of the diminished granular term is not dramatically changed, as we can see in Table 6, because it is relevant only for large countries. In Panel d) of Figure 2, we can see that this is the case: the relative contribution of the granular term becomes relatively smaller in Large Advanced and Emerging economies, while its relative size remains unchanged for Small countries. Among Advanced economies, it is clear now that Small countries suffer relatively more from co-ownership spillovers, just like Emerging economies.

Some countries are important contributors to co-ownership spillovers. We compute a measure of the contribution of countries as the average country allocations in portfolios $\tilde{w}_{k,t}$ multiplied by the volatility of the country-specific expectation residuals $l^i_{k,t}$. Figure 3 shows this measure and contrast it with the country allocations $\tilde{w}_{k,t}$ and with the volatility of $l^i_{k,t}$. First, it appears that the large contributors are mostly countries with large allocations in portfolios. Among emerging economies, those are the BRICs (Brazil, Russian Federation, India, China), but also South Korea and Mexico. Among advanced economies, those are the main G7 countries: UK, the US, France, Japan and Germany. However, the country volatility is not per se a systematic source of contribution. For instance, Argentina, Greece and Venezuela, have volatile expectations but do not contribute to co-ownership spillovers because they constitute a small share of portfolios.

Figure 3: Contributors of co-ownership spillovers

a) b)

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A Extension with multiple funds per investor - Model details

In this Appendix, we solve the model's extension presented in Section 4.6. We focus here on the equivalent of Lemma 4.1 and Proposition 4.1 of the main model. The main text in Section 4.6 focuses on the equivalent of Corollary 4.1.

We first derive the equivalent of Lemma 4.1 with multiple funds per investors:

Lemma A.1 *In the presence of portfolio friction (if $p < 1$), the final share of investor i's wealth invested in country k through mutual fund j, $a^{i,j}_{k,t} = \tilde{w}^{i,j}_{k,t} a^{i,j}_t$, is given by:*

$$\begin{aligned} a^{i,j}_{k,t} =& p\frac{E^i_t(R_{k,t+1}) - r}{\gamma V^i_k} \\ & - p\frac{Cov(R_{k,t+1}, R^{i,j}_{p,k^-,t+1}) - Cov(R_{k,t+1}, \mathcal{R}^{i,j}_{p,j^-,t+1})}{V^i_k} a^{i,j}_t \\ & - p\frac{Cov(R_{k,t+1}, \mathcal{R}^{i,j}_{p,j^-,t+1})}{V^i_k}\left(\sum_{j=1}^{J(i)} a^{i,j}_t\right) \\ & + (1-p)\bar{w}^{i,j}_k a^{i,j}_t \end{aligned} \tag{61}$$

where $V^{i,j}_k = V(R_{k,t+1}) - Cov(R_{k,t+1}, R^{i,j}_{p,k^-,t+1})$, $a^{i,j}_t$, the share of investor i's wealth invested *in fund j is given by*

$$a_t^{i,j} = \frac{E_t^i(R_{p,t+1}^{i,j}) - r}{\gamma V_p^{i,j}} - \frac{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})}{V_p^{i,j}} \left(\sum_{j=1}^{J(i)} a_t^{i,j} \right), \tag{62}$$

and the $\left(\sum_{j=1}^{J(i)} a_t^{i,j}\right)$*, the total share of investor i's wealth invested in equity is given by*

$$\left(\sum_{j=1}^{J(i)} a_t^{i,j} \right) = \frac{E_t^i(\mathcal{R}_{p,t+1}^i) - r}{\gamma V(\mathcal{R}_{p,t+1}^i)} \tag{63}$$

where $V_p^{i,j} = V(R_{p,t+1}^{i,j}) - Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})$*,* $R_{p,k^-,t+1}^{i,j}$ *is the return on fund j's portfolio excluding country k, and* $Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j*})$ *is the covariance between the return of the country k asset and the return of investor i's optimal portfolio that excludes fund j* $\mathcal{R}_{p,j^-,t+1}^{i,j*} = \sum_{j'=1}^{J(i),j'\neq j} \left(\sum_{k\in\mathcal{S}(i,j')} \tilde{w}_{k,t}^{i,j'} R_{k,t+1}\right) a_t^{i,j'} / (\sum_{j=1}^{J(i),j'\neq j} a_t^{i,j})$*.*

Proof. See proof in Appendix B.4. ■

Equation (61) is similar to Equation (16). The last term represent the co-ownership spillovers. The third term is a new term that represents portfolio reallocation spillovers, but at the investor level. The second term is close to the fund-level portfolio reallocation spillovers that we find in the second line of Equation (16), with the nuance that now this term is positively influenced by the covariance between the return in country k and the overall portfolio that excludes fund j. This effect comes from the fact that, for a given total allocation of investor i to equity funds, a higher allocation to fund j implies that investment in the other funds is less attractive. If returns in country k are positively correlated with the returns in these other funds, then some capital is reallocated to country k as k would be a relatively more profitable close substitute to these other funds.

Similarly, if we take into account the fund's optimal setting of the default portfolio shares, we obtain the capital flows as a function of expectations and derive the equivalent of Proposition 4.1 with multiple funds per investor:

Proposition A.1 *We further assume that Assumption 4.1 is satisfied, and that* $\bar{E}^i(a_t^{i,j}) \simeq \bar{a}^{i,j}$ *where we define* $\bar{a}^{i,j}$ *as the share of investor i' wealth invested in fund i that would hold* *under the beginning-of-period information. In that case, Equation* (61) *can be written as:*

$$\begin{aligned}
a_{k,t}^{i,j} =& p\frac{E_t^i(R_{k,t+1}) - r}{\gamma V_k^{i,j}} \\
&+ (1-p)\left(\frac{\bar{E}^i(R_{k,t+1}) - r - \gamma Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})\left(\sum_{j=1}^{J(i)} \bar{E}^i a_t^{i,j}\right)}{\gamma V_k^{i,j} \bar{E}^i(a_t^{i,j})}\right) a_t^{i,j} \\
&- \left(\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j}) - Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_k^{i,j}}\right) a_t^{i,j} \\
&- p\frac{Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_p^{i,j}}\left(\sum_{j=1}^{J(i)} a_t^{i,j}\right)
\end{aligned} \tag{64}$$

Proof. See proof in Appendix B.5. ■

This proposition shows that the portfolio friction does not affect the portfolio reallocation spillovers at the fund level, as in Proposition 4.1, since we can see that the third term of Equation (64) does not depend on p. Indeed, these spillovers arise automatically from the "fixed" part of the portfolio share, which does not depend on expectations. The co-ownership spillovers arise from the ex ante excess return expectation for country k, $\bar{E}^i(R_{k,t+1}) - r$, similarly as before. The last term summarizes the contribution of the total investor's portfolio expectations to the capital flows to country k.

We define

$$\begin{aligned}
\beta_k^{i,j} =& p\left(\frac{1}{\bar{E}^i a_{k,t}^{i,j}}\right)\frac{1}{\gamma V_k^i} \\
\delta_k^{i,j} =& (1-p)\left(\frac{1}{\bar{E}^i a_{k,t}^{i,j}}\right)\left(\frac{\bar{E}^i(R_{k,t+1}) - r - \gamma Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})\left(\sum_{j=1}^{J(i)} \bar{E}^i a_t^{i,j}\right)}{\gamma^2 V_k^{i,j} V_p^{i,j} \bar{E}^i(a_t^{i,j})}\right) \\
&- \left(\frac{1}{\bar{E}^i a_{k,t}^{i,j}}\right)\left(\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j}) - Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{\gamma V_k^{i,j} V_p^{i,j}}\right) \\
\theta_k^{i,j} =& -p\left(\frac{1}{\bar{E}^i a_{k,t}^{i,j}}\right)\frac{Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{\gamma V_k^{i,j} V_p^{i,j}} - \delta_i^k \frac{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})}{V_p^{i,j}}
\end{aligned} \tag{65}$$

Using Proposition A.1, we can then show that capital flows can be decomposed as described in Equation (34) in the main text.

If we define

$$
\begin{aligned}
\eta_k^{i,j} =& (1-p)\left(\frac{1}{\bar{E}^i a_{k,t}^{i,j}}\right)\left(\frac{\bar{E}^i(R_{k,t+1}) - r - \gamma Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})\left(\sum_{j=1}^{J(i)} \bar{E}^i a_t^{i,j}\right)}{\gamma^2 V_k^{i,j} V_p^{i,j} \bar{E}^i(a_t^{i,j})}\right) \\
\phi_k^{i,j} =& \frac{1}{\bar{E}^i a_{k,t}^{i,j} V_k^{i,j}}
\end{aligned}
\tag{66}
$$

then δ can be described by Equation (35).

B Proofs

B.1 Proof of Lemma 4.1

Note that we can now define the expected aggregate equity return for fund i, from the point of view of investor i:

$$
\begin{aligned}
E_t^i(R_{p,t+1}^i) &= [pw_t^{i*} + (1-p)\bar{w}^i]' E_t^i(R_{t+1}) \\
&= \tilde{w}_t^{i'} E_t^i(R_{t+1})
\end{aligned}
\tag{67}
$$

where $\tilde{w}_t^{i'} = pw_t^{i*} + (1-p)\bar{w}^i$. Indeed, when deciding a_t^i, the investor knows w_t^{i*} and $\bar{w}^i$, but does not know which allocation will hold.

Similarly,

$$
\begin{aligned}
V(R_{p,t+1}^i) &= \tilde{w}_t^{i'} V(R_{t+1}) \tilde{w}_t^i \\
&= \tilde{w}_t^{i'} V^R \tilde{w}_t^i
\end{aligned}
\tag{68}
$$

using the independence between the portfolio updating probability and the returns.

The optimal equity investment equation (13), combined with Equations (67) and (68), yields

$$
\tilde{w}_t^{i'} E_t^i(R_{t+1}) - r = \gamma \tilde{w}_t^{i'} V^R \tilde{w}_t^i a_t^i \tag{69}
$$

First consider the case where the fund can update its portfolio, described by Equation

(15). We left-multiply Equation (15) by $\tilde{w}_t^{i'}$ and expand it:

$$
\begin{aligned}
\tilde{w}_t^{i'} E_t^i(R_{t+1}) - E_t^i(R_{k,t+1}) &= \gamma \tilde{w}_t^{i'} (V^R - V_k^R) w_t^{i*} a_t^i \\
&= \gamma \tilde{w}_t^{i'} V^R w_t^{i*} a_t^i - \underbrace{\gamma \tilde{w}_t^{i'} V_k^R w_t^{i*} A_t^i}_{\gamma v_k^R w_t^{i*} a_t^i} \\
&= \underbrace{\gamma \tilde{w}_t^{i'} V^R \tilde{w}_t^i a_t^i}_{\tilde{w}_t^{i'} E_t^i(R_{t+1}) - r} + \gamma \tilde{w}_t^{i'} V^R (w_t^{i*} - \tilde{w}_t^i) a_t^i - \gamma v_k^R w_t^{i*} a_t^i \\
&= \tilde{w}_t^{i'} E_t^i(R_{t+1}) - r + \gamma \tilde{w}_t^{i'} V^R (\underbrace{w_t^{i*} - \tilde{w}_t^i}_{(1-p)(w_t^{i*} - \bar{w}^i)}) a_t^i - \gamma v_k^R w_t^{i*} a_t^i \\
&= \tilde{w}_t^{i'} E_t^i(R_{t+1}) - r + \gamma (1-p) \underbrace{\tilde{w}_t^{i'}}_{\bar{w}^{i'} + p(w_t^{i*'} - \bar{w}^{i'})} V^R (w_t^{i*} - \bar{w}^i) A_t^i - \gamma v_k^R w_t^{i*} A_t^i \\
&= \tilde{w}_t^{i'} E_t^i(R_{t+1}) - r + \gamma(1-p) \bar{w}^{i'} V^R (w_t^{i*} - \bar{w}^i) a_t^i + \gamma p (1-p) (w_t^{i*'} - \bar{w}^{i'}) V^R (w_t^{i*} - \bar{w}^i) a_t^i - \gamma v_k^R w_t^{i*} a_t^i \\
&= \tilde{w}_t^{i'} E_t^i(R_{t+1}) - r + \gamma(1-p) \bar{w}^{i'} V^R (w_t^{i*} - \bar{w}^i) a_t^i + \gamma p (1-p) (w_t^{i*'} - \bar{w}^{i'}) V^R (w_t^{i*} - \bar{w}^i) a_t^i - \gamma v_k^R w_t^{i*} a_t^i
\end{aligned} \tag{70}
$$

Note that the the term $(w_t^{i*'} - \bar{w}^{i'}) V^R (w_t^{i*} - \bar{w}^i)$ is equal to $V(R_{p,t+1}^{i*} - \bar{R}_{p,t+1}^i)$.

Besides, note that the term $\bar{w}^{i'} V^R (w_t^{i*} - \bar{w}^i) = Cov\left(\sum_{k=1}^N \bar{w}_k^i R_{k,t+1}, \sum_{k=1}^N (w_{k,t}^{i*} - \bar{w}_k^i) R_{k,t+1}\right) = \sum_{j=1}^N (w_{j,t}^{i*} - \bar{w}_j^i) \sum_{k=1}^N \bar{w}_k^i Cov(R_{j,t+1}, R_{k,t+1})$ can be approximated by zero. Indeed, the term $\sum_{k=1}^N \bar{w}_k^i Cov(R_{j,t+1}, R_{k,t+1})$ is known in the beginning of period, while $w_{j,t}^{i*} - \bar{w}_k^i$ is a surprise. This means that $w_{j,t}^{i*} - \bar{w}_j^i$ and $\sum_{k=1}^N \bar{w}_k^i Cov(R_{j,t+1}, R_{k,t+1})$ are uncorrelated across countries. Since the $w_{j,t}^{i*} - \bar{w}_j^i$ terms sum to 1, $\sum_{j=1}^N (w_{j,t}^{i*} - \bar{w}_j^i) \sum_{k=1}^N \bar{w}_k^i Cov(R_{j,t+1}, R_{k,t+1})$ should converge to zero as N goes to infinity. We assume that N is large enough to approximate $\bar{w}^{i'} V^R (w_t^{i*} - \bar{w}^i) = 0$.

Therefore, we have

$$
\tilde{w}_t^{i'} E_t^i(R_{t+1}) - E_t^i(R_{k,t+1}) = \tilde{w}_t^{i'} E_t^i(R_{t+1}) - r + \gamma p (1-p) V(R_{p,t+1}^{i*} - \bar{R}_{p,t+1}^i) a_t^i - \gamma v_k^R w_t^{i*} a_t^i \tag{71}
$$

After rearranging this equation, we obtain

$$
\begin{aligned}
w_{k,t}^{i*} a_t^i = & \frac{E_t^i(R_{k,t+1}) - r}{\gamma [V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i*})]} \\
& - \frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i*})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i*})} a_t^i \quad + p(1-p) \frac{V(R_{p,t+1}^{i*} - \bar{R}_{p,t+1}^i)}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i*})} a_t^i
\end{aligned} \tag{72}
$$

where $w^{i*}_{k,t}a^i_t$ is the total flow to country k from investor i if the fund updates its portfolio, and $Cov(R_{k,t+1}, R^{i*}_{p,k^-,t+1}) = Cov(R_{k,t+1}, \sum_{j,j\neq k}^N w^{i*}_{j,t} R_{j,t+1}/(1-w^{i*}_{k,t}))$ is the covariance between the return of the country k asset and the optimal portfolio that excludes k.

Note that $Cov(R_{k,t+1}, R^{i*}_{p,k^-,t+1}) = Cov(R_{k,t+1}, R^{i}_{p,k^-,t+1}) + Cov(R_{k,t+1}, R^{i*}_{p,k^-,t+1} - R^{i}_{p,k^-,t+1})$ and that $Cov(R_{k,t+1}, R^{i*}_{p,k^-,t+1} - R^{i}_{p,k^-,t+1}) = (1-p)Cov(R_{k,t+1}, \bar{R}^{i}_{p,k^-,t+1} - R^{i*}_{p,k^-,t+1}) = (1-p)\sum_{j=1,j\neq k}^N [\bar{w}^i_j/(1-\bar{w}^i_k) - w^{i*}_j/(1-w^{i*}_k)] Cov(R_{k,t+1}, R_{j,t+1})$. The innovations is weights are uncorrelated to the covariance of the country-k return and the other country returns, which are constant terms. This covariance can then be approximated by zero. Therefore, $Cov(R_{k,t+1}, R^{i*}_{p,k^-,t+1}) = Cov(R_{k,t+1}, R^{i}_{p,k^-,t+1})$. This, together with $\tilde{w}^i_{k,t} = pw^{i*}_{k,t} + (1-p)\bar{w}^i_k$, yields Equation (16).

B.2 Proof of Proposition 4.1

We left-multiply (14) by $\bar{w}^{i'}$ and expand it:

$$\begin{aligned}
\bar{w}^{i'}_t \bar{E}^i(R_{t+1}) - \bar{E}^i(R_{k,t+1}) &= \gamma \bar{w}^{i'}(\bar{V}^R - \bar{V}^R_k)\bar{w}^i \bar{E}^i(a^i_t) \\
&= \gamma \bar{w}^{i'}\bar{V}^R \bar{w}^i \bar{E}^i(a^i_t) - \gamma \bar{w}^{i'}\bar{V}^R_k \bar{w}^i \bar{E}^i(a^i_t) \\
&= \gamma \underbrace{\bar{w}^{i'}\bar{V}^R \bar{w}^i}_{\bar{V}(\bar{R}^i_{p,t+1})} \bar{E}^i(a^i_t) - \gamma \underbrace{\bar{w}^{i'}\bar{V}^R_k}_{\bar{v}^R_k} \bar{w}^i \bar{E}^i(a^i_t) \\
&= \gamma \bar{V}(\bar{R}^i_{p,t+1})\bar{E}^i(a^i_t) - \gamma \bar{v}^R_k \bar{w}^i \bar{E}^i(a^i_t)
\end{aligned} \tag{73}$$

We then obtain

$$\underbrace{\bar{w}^{i'}_t \bar{E}^i(R_{t+1})}_{\bar{E}^i(\bar{R}^i_{p,t+1})} - \bar{E}^i(R_{k,t+1}) = \gamma \bar{V}(\bar{R}^i_{p,t+1})\bar{E}^i(a^i_t) - \gamma \bar{v}^R_k \bar{w}^i \bar{E}^i(a^i_t) \tag{74}$$

We define $\bar{a}^i$ such that

$$\bar{a}^i = \frac{\bar{E}^i(\bar{R}^i_{p,t+1}) - r}{\gamma \bar{V}(\bar{R}^i_{p,t+1})} \tag{75}$$

$\bar{a}^i$ is the investments share to fund i that would be consistent with the beginning-of-period information $\bar{\mathcal{I}}^i$. Note that this is not necessarily equal to $\bar{E}^i(a^i_t)$, the expected share conditional on $\bar{\mathcal{I}}^i$, which should satisfy

$$\bar{E}^i(a^i_t) = \bar{E}^i\left(\frac{E^i_t(R^i_{p,t+1}) - r}{\gamma V(R^i_{p,t+1})}\right) \tag{76}$$

We can therefore write

$$\gamma \bar{v}_k^R \bar{w}^i \bar{E}^i(a_t^i) = -\underbrace{\left[\bar{E}^i(\bar{R}_{p,t+1}^i) - r\right]}_{\gamma \bar{V}(\bar{R}_{p,t+1}^i)\bar{A}^i} + \left[\bar{E}^i(R_{k,t+1}) - r\right] + \gamma \bar{V}(\bar{R}_{p,t+1}^i)\bar{E}^i(a_t^i) \tag{77}$$

$$= \left(\bar{E}^i(R_{k,t+1}) - r + \gamma \bar{V}(\bar{R}_{p,t+1}^i)\left[\bar{E}^i(a_t^i) - \bar{a}^i\right]\right) \tag{78}$$

After rearranging this equation, we obtain

$$\begin{aligned}\bar{w}_k^i \bar{E}^i(a_t^i) =& \frac{\bar{E}^i(R_{k,t+1}) - r + \gamma \bar{V}(\bar{R}_{p,t+1}^i)\left[\bar{E}^i(a_t^i) - \bar{a}^i\right]}{\gamma[\bar{V}(R_{k,t+1}) - \bar{Cov}(R_{k,t+1}, \bar{R}_{p,k^-,t+1}^i)]} \\ &- \frac{\bar{Cov}(R_{k,t+1}, \bar{R}_{p,k^-,t+1}^i)}{\bar{V}(R_{k,t+1}) - \bar{Cov}(R_{k,t+1}, \bar{R}_{p,k^-,t+1}^i)}\bar{E}^i(a_t^i)\end{aligned} \tag{79}$$

where $\bar{Cov}(R_{k,t+1}, \bar{R}_{p,k^-,t+1}^i) = \bar{Cov}(R_{k,t+1}, \sum_{j,j\neq k}^N \bar{w}_j^i R_{j,t+1}/(1 - \bar{w}_k^i))$ is the covariance between the return of the country k asset and the predetermined portfolio that excludes k.

We then multiply both sides of this equation by $a_t^i/\bar{E}^i(a_t^i)$:

$$\begin{aligned}\bar{w}_k^i a_t^i =& \frac{\bar{E}^i(R_{k,t+1}) - r + \gamma \bar{V}(\bar{R}_{p,t+1}^i)\left[\bar{E}^i(a_t^i) - \bar{a}^i\right]}{\gamma[\bar{V}(R_{k,t+1}) - \bar{Cov}(R_{k,t+1}, \bar{R}_{p,k^-,t+1}^i)]}\frac{a_t^i}{\bar{E}^i(a_t^i)} \\ &- \frac{\bar{Cov}(R_{k,t+1}, \bar{R}_{p,k^-,t+1}^i)}{\bar{V}(R_{k,t+1}) - \bar{Cov}(R_{k,t+1}, \bar{R}_{p,k^-,t+1}^i)}a_t^i\end{aligned} \tag{80}$$

We multiply Equation (72) by p and Equation (80) by $1-p$ and sum both equations. We then obtain

$$\begin{aligned}\tilde{w}_k^i a_t^i =& p\frac{E_t^i(R_{k,t+1}) - r}{\gamma[V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)]} + (1-p)\frac{\bar{E}^i(R_{k,t+1}) - r}{\gamma[\bar{V}(R_{k,t+1}) - \bar{Cov}(R_{k,t+1}, R_{p,k^-,t+1}^i)]}\frac{a_t^i}{\bar{E}^i(a_t^i)} \\ &+ (1-p)\gamma\frac{\bar{V}(\bar{R}_{p,t+1}^i)}{\gamma[\bar{V}(R_{k,t+1}) - \bar{Cov}(R_{k,t+1}, R_{p,k^-,t+1}^i)]}\frac{\bar{E}^i(a_t^i) - \bar{a}^i}{\bar{E}^i(a_t^i)}a_t^i \\ &- \left(p\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)} + (1-p)\frac{\bar{Cov}(R_{k,t+1}, R_{p,k^-,t+1}^i)}{\bar{V}(R_{k,t+1}) - \bar{Cov}(R_{k,t+1}, R_{p,k^-,t+1}^i)}\right)a_t^i \\ &+ p^2(1-p)\frac{V(R_{p,t+1}^{i*} - \bar{R}_{p,t+1}^i)}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^i)}a_t^i\end{aligned} \tag{81}$$

where a similar argument as before has been applied to show that $\bar{Cov}(R_{k,t+1}, \bar{R}_{p,k^-,t+1}^i) = \bar{Cov}(R_{k,t+1}, R_{p,k^-,t+1}^i)$.

Under Assumption 4.1, we would have

$$p\frac{Cov(R_{k,t+1},R^i_{p,k^-,t+1})}{V(R_{k,t+1})-Cov(R_{k,t+1},R^i_{p,k^-,t+1})}+(1-p)\frac{\bar{Cov}(R_{k,t+1},R^i_{p,k^-,t+1})}{\bar{V}(R_{k,t+1})-\bar{Cov}(R_{k,t+1},R^i_{p,k^-,t+1})}$$
$$=p\frac{Cov(R_{k,t+1},R^i_{p,k^-,t+1})}{V(R_{k,t+1})-Cov(R_{k,t+1},R^i_{p,k^-,t+1})}+(1-p)\frac{\kappa Cov(R_{k,t+1},R^i_{p,k^-,t+1})}{\kappa V(R_{k,t+1})-\kappa Cov(R_{k,t+1},R^i_{p,k^-,t+1})}$$
$$=\frac{Cov(R_{k,t+1},R^i_{p,k^-,t+1})}{V(R_{k,t+1})-Cov(R_{k,t+1},R^i_{p,k^-,t+1})} \quad (82)$$

Then, assuming $\bar{E}^i(a^i_t)-\bar{a}^i\simeq 0$, we obtain Equation (17).

B.3 Proof of Corollary 4.1

Consider β^i_k and δ^i_k as defined in equation (21). β^i_k is increasing in p and δ^i_k is decreasing in p.

Now consider $\beta^i_k+\delta^i_k$:

$$\beta^i_k+\delta^i_k=\frac{1}{\gamma V^i_k\bar{E}^i(a^i_{k,t})}\left[p+(1-p)\frac{\left[\bar{E}^i(R_{k,t+1})-r\right]}{\gamma V^i_p\bar{E}^i(a^i_t)}-\frac{Cov(R_{k,t+1},R^i_{p,k^-,t+1})}{V^i_p}\right]$$
$$=\frac{1}{\gamma V^i_k\bar{E}^i(a^i_{k,t})}\left[p+(1-p)\frac{\left[\bar{E}^i(R_{k,t+1})-r\right]}{\left[\bar{E}^i(R^i_{p,t+1})-r\right]}-\frac{Cov(R_{k,t+1},R^i_{p,k^-,t+1})}{V^i_p}\right] \quad (83)$$

where we used (13). If $\bar{E}^i(R_{k,t+1})=\bar{E}^i(R^i_{p,t+1})$, then

$$\beta^i_k+\delta^i_k=\frac{1}{\gamma V^i_k\bar{E}^i(a^i_{k,t})}\left[1-\frac{Cov(R_{k,t+1},R^i_{p,k^-,t+1})}{V^i_p}\right] \quad (84)$$

which is independent of p.

B.4 Proof of Lemma A.1

We now derive a more general version of Equation (69):

$$\tilde{W}^{i\prime}_t E^i_t(R_{t+1})-r=\gamma\tilde{W}^{i\prime}_t V^R\tilde{W}^i_t a^i_t \quad (85)$$

where $\tilde{W}^i_t=(\tilde{w}^{i,1}_t,..,\tilde{w}^{i,j}_t,..,\tilde{w}^{i,N_i}_t)$ is the matrix that collects the average portfolio weights of each individual investors and $a^{i\prime}_t=(a^{i,1}_t,..,a^{i,j}_t,..,a^{i,N_i}_t)'$ is the vector that collects the share of investor i's investment in each fund j. Note that, because each fund j invests in a limited set of countries $\mathcal{S}(i,j)$, some of the weights may be equal to zero. We have $\tilde{w}_t^{i,j} = pw_t^{i,j*} + (1-p)\bar{w}^{i,j}$ for all (i,j).

Updating funds will set their portfolio shares as follows:

$$Id(i,j)\left[E_t^i(R_{t+1}) - E_t^i(R_{k,t+1})\right] = \gamma Id(i,j)(V^R - V_k^R)W_t^{i*}a_t^i \tag{86}$$

where the k^{th} element of the diagonal of $Id(i,j)$ is equal to one if $k \in \mathcal{S}(i,j)$, and zero otherwise. For $k \notin \mathcal{S}(i,j)$, $w_t^{i,j*} = 0$. Therefore, $w^{i,j*\prime}Id(i,j) = w^{i,j*\prime}$.

The country allocation by passive funds, $\bar{W}^i$, is characterized as follows:

$$Id(i,j)\left[\bar{E}(R_{t+1}) - \bar{E}(R_{k,t+1}) = \gamma Id(i,j)(V^R - V_k^R)\bar{W}^i\bar{E}^i(a_t^i)\right] \tag{87}$$

where $\bar{E}^i(a_t^i)$ is defined by

$$\bar{E}^i(a_t^i) = \bar{E}^i\left(\left(\gamma\tilde{W}_t^{i\prime}V^R\tilde{W}_t^i\right)^{-1}\left(\tilde{W}_t^{i\prime}E_t^i(R_{t+1}) - r\right)\right) \tag{88}$$

For $k \notin \mathcal{S}(i,j)$, $\bar{w}^{i,j} = 0$. Therefore, $\bar{w}^{i,j\prime}Id(i,j) = \bar{w}^{i,j\prime}$.

In Equation (85), we focus on the j^{th} line:

$$w_t^{i,j\prime}E_t^i(R_{t+1}) - R - \gamma w_t^{i,j\prime}V^R W_t^i A_t^i = 0 \tag{89}$$

We left-multiply (86) by $\tilde{w}_t^{i,j\prime}$ to obtain

$$\tilde{w}_t^{i,j\prime}Id(i,j)[E_t^i(R_{t+1}) - E_t^i(R_{k,t+1})] = \gamma\tilde{w}_t^{i,j\prime}Id(i,j)(V^R - V_k^R)W_t^{i*}a_t^i \tag{90}$$

Using $\tilde{w}_t^{i,j\prime}Id(i,j) = \tilde{w}_t^{i,j\prime}$, we get

$$\begin{aligned}
\tilde{w}_t^{i,j\prime}[E_t^i(R_{t+1}) - E_t^i(R_{k,t+1})] &= \gamma\tilde{w}_t^{i,j\prime}(V^R - V_k^R)W_t^{i*}a_t^i \\
\tilde{w}_t^{i,j\prime}E_t^i(R_{t+1}) - E_t^i(R_{k,t+1}) &= \gamma\tilde{w}_t^{i,j\prime}V^R W_t^{i*}a_t^i - \gamma\underbrace{\tilde{w}_t^{i,j\prime}V_k^R}_{v_k^R}W_t^{i*}a_t^i \\
&= \underbrace{\gamma\tilde{w}_t^{i,j\prime}V^R W_t^i a_t^i}_{\tilde{w}_t^{i,j\prime}E_t^i(R_{t+1}) - r} + (1-p)\gamma\tilde{w}_t^{i,j\prime}V^R(W_t^{i*} - \bar{W}^i)a_t^i - \gamma v_k^R W_t^{i*}a_t^i \\
&= \tilde{w}_t^{i,j\prime}E_t^i(R_{t+1}) - r + (1-p)\gamma\bar{w}^{i,j\prime}V^R(W_t^{i*} - \bar{W}^i)a_t^i \\
&+ p(1-p)\gamma(w_t^{i,j*\prime} - \bar{w}^{i,j\prime})V^R(W_t^{i*} - \bar{W}^i)a_t^i + \gamma v_k^R W_t^{i*}a_t^i
\end{aligned} \tag{91}$$

Note that the the term $(w_t^{i,j*\prime} - \bar{w}^{i,j\prime})V^R(W_t^{i*} - \bar{W}^i)a_t^i$ is equal to $Cov(R_{p,t+1}^{i,j*} - \bar{R}_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i*} - \bar{\mathcal{R}}_{p,t+1}^i)\sum_{j=1}^{J(i)} a_t^{i,j}$, where $\mathcal{R}_{p,t+1}^i = \sum_{j=1}^{J(i)}(\sum_{k\in\mathcal{S}(i,j)}\tilde{w}_{k,t}^{i,j}R_{k,t+1})a_t^{i,j}/(\sum_{j=1}^{J(i)} a_t^{i,j})$ refers to the conditional returns of the whole equity portfolio of investor i.

Besides, note that the term $\bar{w}^{i,j'}V^R(W_t^{i*} - \bar{W}^i)$ can be approximated by zero, using a similar arguments as in the proof of Lemma 4.1.

Therefore, we have

$$\tilde{w}_t^{i'}E_t^i(R_{t+1}) - E_t^i(R_{k,t+1}) = \tilde{w}_t^{i,j'}E_t^i(R_{t+1}) - r + \gamma v_k^R W_t^{i*} a_t^i + p(1-p)\gamma Cov(R_{p,t+1}^{i,j*} - \bar{R}_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i*} - \bar{\mathcal{R}}_{p,t+1}^{i}) \left(\sum_{j=1}^{J(i)} a_t^{i,j}\right) \quad (92)$$

After rearranging this equation, we obtain

$$\begin{aligned} w_{k,t}^{i,j*}a_t^{i,j} =& \frac{E_t^i(R_{k,t+1}) - r}{\gamma[V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j*})]} \\ &- \frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j*})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j*})} a_t^{i,j} \\ &- \frac{Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j*})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j*})} \left(\sum_{j'=1}^{J(i),j'\neq j} a_t^{i,j'}\right) \\ &+ p(1-p)\frac{V(R_{p,t+1}^{i,j*} - \bar{R}_{p,t+1}^{i,j})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j*})} \left(\sum_{j=1}^{J(i)} a_t^{i,j}\right) \end{aligned} \quad (93)$$

where $w_{k,t}^{i,j*}a_t^{i,j}$ is the total flow to country k from investor i if the fund updates its portfolio, $Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j*}) = Cov(R_{k,t+1}, \sum_{j,j\neq k}^{N} w_{j,t}^{i*} R_{j,t+1}/(1-w_{k,t}^{i*}))$ is the covariance between the return of the country k asset and the optimal fund j portfolio that excludes country k, and $Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j*}) = Cov(R_{k,t+1}, \sum_{j'=1}^{J(i),j'\neq j} \left(\sum_{k\in\mathcal{S}(i,j')} \tilde{w}_{k,t}^{i,j'} R_{k,t+1}) a_t^{i,j'} / (\sum_{j=1}^{J(i)} a_t^{i,j}\right)$ is the covariance between the return of the country k asset and the optimal investor i portfolio that excludes fund j.

Using a similar arguments as in the proof of Lemma 4.1, we argue that $Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j*}) = Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j})$ and $Cov(R_{k,t+1}, R_{p,j^-,t+1}^{i,j*}) = Cov(R_{k,t+1}, R_{p,j^-,t+1}^{i,j})$. This, together with $\tilde{w}_{k,t}^{i,j} = pw_{k,t}^{i,j*} + (1-p)\bar{w}_k^{i,j}$, yields

$$\begin{aligned} w_{k,t}^{i,j*} a_t^{i,j} = & \frac{E_t^i(R_{k,t+1}) - r}{\gamma[V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j})]} \\ & - \frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j})} a_t^{i,j} \\ & - \frac{Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j})} \left(\sum_{j'=1}^{J(i),j' \neq j} a_t^{i,j'} \right) \\ & + p(1-p) \frac{V(R_{p,t+1}^{i,j*} - \bar{R}_{p,t+1}^{i,j})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j})} \left(\sum_{j=1}^{J(i)} a_t^{i,j} \right) \end{aligned} \tag{94}$$

This yields Equation (61), by applying .

By left-multiplying Equation (85) by $\frac{a_t^{i'}}{\sum_{j=1}^{J(i)} a_t^{i,j}}$, we can show that the total share allocated to equity $\left(\sum_{j=1}^{J(i)} a_t^{i,j}\right)$ must satisfy

$$\begin{aligned} \left(\sum_{j=1}^{J(i)} a_t^{i,j} \right) &= \left(\gamma \frac{a_t^{i'}}{\sum_{j=1}^{J(i)} a_t^{i,j}} \tilde{W}_t^{i'} V^R \tilde{W}_t^i \frac{a_t^i}{\sum_{j=1}^{J(i)} a_t^{i,j}} \right)^{-1} \left(\frac{a_t^{i'}}{\sum_{j=1}^{J(i)} a_t^{i,j}} \tilde{W}_t^{i'} E_t^i(R_{t+1}) - r \right) \\ &= \frac{E_t^i(\mathcal{R}_{p,t+1}^i) - r}{\gamma V(\mathcal{R}_{p,t+1}^i)} \end{aligned} \tag{95}$$

This yields Equation (63).

We can also derive $a_t^{i,j}$. To do so, we focus on the j^{th} line of Equation (85):

$$\begin{aligned} \underbrace{\tilde{w}_t^{i,j'} E_t^i(R_{t+1})}_{E^i(R_{p,t+1}^{i,j})} - r &= \gamma \tilde{w}_t^{i,j'} V^R \tilde{W}_t^i a_t^i \\ &= \gamma \underbrace{\tilde{w}_t^{i,j'} V^R \tilde{w}_t^{i,j}}_{V(R_{p,t+1}^{i,j})} a_t^{i,j} + \gamma \underbrace{\tilde{w}_t^{i,j'} V^R \tilde{W}_t^{i,j^-} a_t^{i,j^-}}_{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})\left(\sum_{j'=1}^{J(i),j' \neq j} a_t^{i,j'}\right)} \end{aligned} \tag{96}$$

where $\tilde{W}_t^{i,j^-}$ contains all the columns of $\tilde{W}_t^{i,j}$ except $\tilde{w}_t^{i,j}$ and a_t^{i,j^-} contains all the elements of a_t^i except $a_t^{i,j}$. This yields

$$a_t^{i,j} = \frac{E_t^i(R_{p,t+1}^{i,j}) - r}{\gamma[V(R_{p,t+1}^{i,j}) - Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})]} - \frac{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})}{V(R_{p,t+1}^{i,j}) - Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})} \left(\sum_{j=1}^{J(i)} a_t^{i,j} \right) \tag{97}$$

This yields Equation (62).

B.5 Proof of Proposition A.1

We left-multiply (87) by $\bar{w}^{i,j'}$ and expand it:

$$\begin{aligned} \bar{w}_t^{i,j'}\bar{E}^i(R_{t+1}) - \bar{E}^i(R_{k,t+1}) &= \gamma\bar{w}^{i,j'}(\bar{V}^R - \bar{V}_k^R)\bar{W}^i\bar{E}^i(a_t^i) \\ &= \gamma\bar{w}^{i,j'}\bar{V}^R\bar{W}^i\bar{E}^i(a_t^i) - \gamma\bar{w}^{i,j'}\bar{V}_k^R\bar{W}^i\bar{E}^i(a_t^i) \end{aligned} \tag{98}$$

and note that

$$\gamma\bar{w}^{i,j'}\bar{V}^R\bar{W}^i\bar{E}^i(a_t^i) = \gamma\underbrace{\bar{w}^{i,j'}V^R\bar{w}^{i,j}}_{\bar{V}(\bar{R}_{p,t+1}^{i,j})}\bar{E}^i(a_t^{i,j}) + \gamma\underbrace{\bar{w}^{i,j'}V^R\bar{W}_t^{i,j^-}\bar{E}^i(a_t^{i,j^-})}_{\bar{C}ov(\bar{R}_{p,t+1}^{i,j},\mathcal{R}_{p,t+1}^{i,j^-})\left(\sum_{j'=1}^{J(i),j'\neq j}\bar{E}^i(a_t^{i,j'})\right)} \tag{99}$$

We then obtain

$$\underbrace{\bar{w}_t^{i,j'}\bar{E}^i(R_{t+1})}_{\bar{E}^i(\bar{R}_{p,t+1}^{i,j})} - \bar{E}^i(R_{k,t+1}) = \gamma\bar{V}(\bar{R}_{p,t+1}^{i,j})\bar{E}^i(a_t^{i,j}) + \gamma\bar{C}ov(\bar{R}_{p,t+1}^{i,j},\mathcal{R}_{p,t+1}^{i,j^-})\left(\sum_{j'=1}^{J(i),j'\neq j}\bar{E}^i(a_t^{i,j'})\right) - \gamma\bar{v}_k^R\bar{W}^i\bar{E}^i(a_t^i) \tag{100}$$

We define $\bar{a}^i$ such that

$$\bar{a}^i = \left(\gamma\bar{W}^{i'}V^R\bar{W}^i\right)^{-1}\left(\bar{W}^{i'}\bar{E}^i(R_{t+1}) - r\right) \tag{101}$$

$\bar{a}^i$ is the investments share to fund i that would be consistent with the beginning-of-period information $\bar{\mathcal{I}}^i$. Note that this is not necessarily equal to $\bar{E}^i(a_t^i)$, the expected share conditional on $\bar{\mathcal{I}}^i$, which should satisfy (88). rom this equation, we can infer $\bar{a}^{i,j}$:

$$\bar{a}^{i,j} = \frac{\bar{E}^i(\bar{R}_{p,t+1}^{i,j}) - r}{\gamma[\bar{V}(\bar{R}_{p,t+1}^{i,j}) - \bar{C}ov(\bar{R}_{p,t+1}^{i,j},\bar{\mathcal{R}}_{p,t+1}^{i,j^-})]} - \frac{\bar{C}ov(\bar{R}_{p,t+1}^{i,j},\bar{\mathcal{R}}_{p,t+1}^{i,j^-})}{V(\bar{R}_{p,t+1}^{i,j}) - \bar{C}ov(\bar{R}_{p,t+1}^{i,j},\bar{\mathcal{R}}_{p,t+1}^{i,j^-})}\left(\sum_{j=1}^{J(i)}\bar{a}_t^{i,j}\right) \tag{102}$$

We can therefore replace $\bar{E}^i(\bar{R}^{i,j}_{p,t+1}) - r$ in Equation (100) and write

$$\gamma\bar{v}^R_k \bar{W}^i \bar{E}^i(a^i_t) = -\gamma[\bar{V}(\bar{R}^{i,j}_{p,t+1}) - \bar{C}ov(\bar{R}^{i,j}_{p,t+1}, \bar{\mathcal{R}}^{i,j^-}_{p,t+1})]\bar{a}^{i,j} - \gamma\bar{C}ov(\bar{R}^{i,j}_{p,t+1}, \bar{\mathcal{R}}^{i,j^-}_{p,t+1}) \left(\sum_{j=1}^{J(i)} \bar{a}^{i,j}_t\right)$$
$$+ \left[\bar{E}^i(R_{k,t+1}) - r\right] + \gamma\bar{V}(\bar{R}^{i,j}_{p,t+1})\bar{E}^i(a^{i,j}_t) + \gamma\bar{C}ov(\bar{R}^{i,j}_{p,t+1}, \bar{\mathcal{R}}^{i,j^-}_{p,t+1}) \left(\sum_{j'=1}^{J(i),j'\neq j} \bar{E}^i(a^{i,j'}_t)\right) \tag{103}$$

$$= \bar{E}^i(R_{k,t+1}) - r + \gamma[\bar{V}(\bar{R}^{i,j}_{p,t+1}) - \bar{C}ov(\bar{R}^{i,j}_{p,t+1}, \bar{\mathcal{R}}^{i,j^-}_{p,t+1})]\left[\bar{E}^i(a^{i,j}_t) - \bar{a}^{i,j}\right]$$
$$+ \gamma\bar{C}ov(\bar{R}^{i,j}_{p,t+1}, \bar{\mathcal{R}}^{i,j^-}_{p,t+1}) \left[\left(\sum_{j=1}^{J(i)} \bar{E}^i(a^{i,j}_t)\right) - \left(\sum_{j=1}^{J(i)} \bar{a}^{i,j}_t\right)\right] \tag{104}$$

Assuming that $\bar{E}^i(a^i_t) \simeq \bar{a}^i_t$ and after rearranging this equation, we obtain

$$\bar{w}^{i,j}_k \bar{E}^i(a^{i,j}_t) = \frac{\bar{E}^i(R_{k,t+1}) - r}{\gamma[\bar{V}(\bar{R}_{k,t+1}) - \bar{C}ov(\bar{R}_{k,t+1}, \bar{R}^{i,j}_{p,k^-,t+1})]}$$
$$- \frac{\bar{C}ov(\bar{R}_{k,t+1}, \bar{R}^{i,j}_{p,k^-,t+1}) - \bar{C}ov(\bar{R}_{k,t+1}, \bar{\mathcal{R}}^{i,j}_{p,j^-,t+1})}{\bar{V}(\bar{R}_{k,t+1}) - \bar{C}ov(\bar{R}_{k,t+1}, \bar{R}^{i,j}_{p,k^-,t+1})} \bar{E}^i(a^{i,j}_t)$$
$$- \frac{\bar{C}ov(\bar{R}_{k,t+1}, \bar{\mathcal{R}}^{i,j}_{p,j^-,t+1})}{\bar{V}(\bar{R}_{k,t+1}) - \bar{C}ov(\bar{R}_{k,t+1}, \bar{R}^{i,j}_{p,k^-,t+1})} \left(\sum_{j=1}^{J(i)} \bar{E}^i(a^{i,j}_t)\right) \tag{105}$$

We then multiply both sides of this equation by $a^{i,j}_t / \bar{E}^i(a^{i,j}_t)$:

$$\bar{w}^{i,j}_k a^{i,j}_t = \frac{\bar{E}^i(R_{k,t+1}) - r}{\gamma[\bar{V}(\bar{R}_{k,t+1}) - \bar{C}ov(\bar{R}_{k,t+1}, \bar{R}^{i,j}_{p,k^-,t+1})]} \frac{a^{i,j}_t}{\bar{E}^i(a^{i,j}_t)}$$
$$- \frac{\bar{C}ov(\bar{R}_{k,t+1}, \bar{R}^{i,j}_{p,k^-,t+1}) - \bar{C}ov(\bar{R}_{k,t+1}, \bar{\mathcal{R}}^{i,j}_{p,j^-,t+1})}{\bar{V}(\bar{R}_{k,t+1}) - \bar{C}ov(\bar{R}_{k,t+1}, \bar{R}^{i,j}_{p,k^-,t+1})} a^{i,j}_t$$
$$- \frac{\bar{C}ov(\bar{R}_{k,t+1}, \bar{\mathcal{R}}^{i,j}_{p,j^-,t+1})}{\bar{V}(\bar{R}_{k,t+1}) - \bar{C}ov(\bar{R}_{k,t+1}, \bar{R}^{i,j}_{p,k^-,t+1})} \left(\sum_{j=1}^{J(i)} \bar{E}^i(a^{i,j}_t)\right) \frac{a^{i,j}_t}{\bar{E}^i(a^{i,j}_t)} \tag{106}$$

We multiply Equation (94) by p and Equation (106) by $1 - p$ and sum both equations.

Using Assumption 4.1, we obtain

$$\begin{aligned}
a^{i,j}_{k,t} =& p\frac{E^i_t(R_{k,t+1}) - r}{\gamma V^{i,j}_k} \\
&- \left(\frac{Cov(R_{k,t+1}, R^{i,j}_{p,k^-,t+1}) - Cov(R_{k,t+1}, \mathcal{R}^{i,j}_{p,j^-,t+1})}{V^{i,j}_k}\right) a^{i,j}_t \\
&+ (1-p)\left(\frac{\bar{E}^i(R_{k,t+1}) - r - \gamma Cov(R_{k,t+1}, \mathcal{R}^{i,j}_{p,j^-,t+1})\left(\sum_{j=1}^{J(i)} \bar{E}^i a^{i,j}_t\right)}{\gamma V^{i,j}_k \bar{E}^i(a^{i,j}_t)}\right) a^{i,j}_t \\
&- p\frac{Cov(R_{k,t+1}, \mathcal{R}^{i,j}_{p,j^-,t+1})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R^{i,j}_{p,k^-,t+1})}\left(\sum_{j=1}^{J(i)} a^{i,j}_t\right) \\
&+ p^2(1-p)\frac{V(R^{i,j*}_{p,t+1} - \bar{R}^{i,j}_{p,t+1})}{V(R_{k,t+1}) - Cov(R_{k,t+1}, R^{i,j}_{p,k^-,t+1})}\left(\sum_{j=1}^{J(i)} a^{i,j}_t\right)
\end{aligned} \tag{107}$$

This yields Equation (64).

B.6 Proof of Corollary 4.2

Consider $\beta^{i,j}_k$ and $\delta^{i,j}_k$ as defined in equation (65). $\beta^{i,j}_k$ is increasing in p and $\delta^{i,j}_k$ is decreasing in p.

Now consider $\beta^{i,j}_k + \delta^{i,j}_k + \theta^{i,j}_k$. After rearranging, we get

$$\begin{aligned}
\beta^{i,j}_k + \delta^{i,j}_k + \theta^{i,j}_k =& \left(\frac{1}{\bar{E}^i a^{i,j}_{k,t} \gamma V^i_k}\right)\left[p\left(1 - \frac{Cov(R_{k,t+1}, \mathcal{R}^{i,j}_{p,j^-,t+1})}{V^{i,j}_p}\right)\right. \\
&+(1-p)\left(\frac{\bar{E}^i(R_{k,t+1}) - r - \gamma Cov(R_{k,t+1}, \mathcal{R}^{i,j}_{p,j^-,t+1})\left(\sum_{j=1}^{J(i)} \bar{E}^i a^{i,j}_t\right)}{\gamma V^{i,j}_p \bar{E}^i(a^{i,j}_t)}\right)\left(1 - \frac{Cov(R^{i,j}_{p,t+1}, \mathcal{R}^{i,j^-}_{p,t+1})}{V^{i,j}_p}\right) \\
&\left.- \left(\frac{Cov(R_{k,t+1}, R^{i,j}_{p,k^-,t+1}) - Cov(R_{k,t+1}, \mathcal{R}^{i,j}_{p,j^-,t+1})}{V^{i,j}_p}\right)\left(1 - \frac{Cov(R^{i,j}_{p,t+1}, \mathcal{R}^{i,j^-}_{p,t+1})}{V^{i,j}_p}\right)\right]
\end{aligned} \tag{108}$$

Using Equation (85), we obtain

$$\beta_k^{i,j} + \delta_k^{i,j} + \theta_k^{i,j} = \left(\frac{1}{\bar{E}^i a_{k,t}^{i,j} \gamma V_k^i}\right) \Bigg[p \left(1 - \frac{Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_p^{i,j}}\right)$$
$$+(1-p)\left(\frac{\bar{E}^i(R_{k,t+1}) - r - \gamma Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})\left(\sum_{j=1}^{J(i)} \bar{E}^i a_t^{i,j}\right)}{\bar{E}^i(R_{p,t+1}^{i,j}) - r - \gamma Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})\left(\sum_{j=1}^{J(i)} \bar{E}^i a_t^{i,j}\right)}\right)\left(1 - \frac{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})}{V_p^{i,j}}\right)$$
$$-\left(\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j}) - Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_p^{i,j}}\right)\left(1 - \frac{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})}{V_p^{i,j}}\right)\Bigg] \tag{109}$$

If $\bar{E}^i(R_{k,t+1}) = \bar{E}^i(R_{p,t+1}^{i,j})$ and $Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j}) = Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})$ as Assumption 4.3 states, then

$$\beta_k^{i,j} + \delta_k^{i,j} + \theta_k^{i,j} =$$
$$\left(\frac{1}{\bar{E}^i a_{k,t}^{i,j} \gamma V_k^i}\right)\left(1 - \frac{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})}{V_p^{i,j}}\right)\left[1 - \left(\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j}) - Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_p^{i,j}}\right)\right] \tag{110}$$

which is independent of p.

Finally, we can write $\theta_k^{i,j}$:

$$\theta_k^{i,j} = \left(\frac{1}{\gamma V_k^{i,j} \bar{E}^i a_{k,t}^{i,j}}\right)\Bigg[- p \frac{Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_p^{i,j}}$$
$$- \frac{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})}{V_p^{i,j}}\Bigg((1-p)\left(\frac{\bar{E}^i(R_{k,t+1}) - r - \gamma Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})\left(\sum_{j=1}^{J(i)} \bar{E}^i a_t^{i,j}\right)}{\gamma V_p^{i,j} \bar{E}^i(a_t^{i,j})}\right)$$
$$-\left(\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j}) - Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_p^{i,j}}\right)\Bigg)\Bigg] \tag{111}$$

Again, we use Equation (85) and Assumption 4.3 and show that

$$\theta_k^{i,j} = -\left(\frac{1}{\gamma V_k^{i,j} \bar{E}^i a_{k,t}^{i,j}}\right) \frac{Cov(R_{p,t+1}^{i,j}, \mathcal{R}_{p,t+1}^{i,j^-})}{V_p^{i,j}}\left[1 - \left(\frac{Cov(R_{k,t+1}, R_{p,k^-,t+1}^{i,j}) - Cov(R_{k,t+1}, \mathcal{R}_{p,j^-,t+1}^{i,j})}{V_p^{i,j}}\right)\right] \tag{112}$$

which is independent of p.